H_o: μ=54 *cm* $\qquad \sigma_{\bar{X}} = \frac{4.5\ cm}{\sqrt{9}} = 1.5\ cm$

H_a: μ>54 (μ_a = 58) $\qquad$ α = .05

<u>4 Steps for finding the Power in a test of hypotheses</u>

1) Write the RR for H_o in terms of z-scores:

2) Write the RR for H_o in terms of $\bar{X}$:

3) Find the probability of a Type II error <u>if μ=58</u>

4) Power = 1 – P(Type II Error):

<u>Probability Distributions for Random Variables (RVs)</u>

- Probability Distribution (for a *discrete* RV) – represented graphically as a <u>*Probability Histogram*</u>
  - Indicates the possible values for a RV
  - Indicates how to assign probabilities for the possible values: $p(x)$ = P(X = x)
- Probability Distribution (for a *continuous* RV) – represented as a <u>*Probability Density Curve*</u>
  - Areas under a smooth curve, $f(x)$, indicate probabilities of values in a given range

<u>Expected Value of a Random Variable</u>

- a weighted average of all possible values for *X*, weighted by the probability of each value
  $E(X) = \mu$, the mean for the RV
  - $E(X) = \sum_{x=-\infty}^{\infty} x\,p(x)$ for a *discrete* RV
  - $E(X) = \int_{x=-\infty}^{\infty} x\,f(x)\,dx$ for a *continuous* RV

<u>Example (*discrete* RV)</u>:
Y = # heads in two tosses of a fair coin

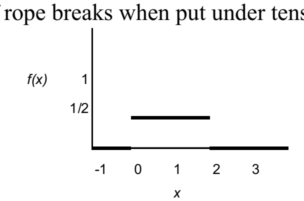
P(Y = 0) = ¼
P(Y = 1) = ½
P(Y = 2) = ¼ and zero otherwise

$$E(Y) = 0 \bullet P(Y=0) + 1 \bullet P(Y=1) + 2 \bullet P(Y=2)$$

$$= 0 \bullet \frac{1}{4} + 1 \bullet \frac{1}{2} + 2 \bullet \frac{1}{4} = 1$$

<u>Example (*continuous* RV)</u>:
X = the position at which a two-meter with length of rope breaks when put under tension (assuming every point is equally likely)

$f(x)$= ½ $0 \le x \le 2$, zero otherwise



$$E(X) = \int_0^2 x\left(\frac{1}{2}\right)dx = \frac{1}{4}x^2\Big|_0^2 = 1 - 0 = 1$$

<u>Properties of Expectation</u>
E(a) =a
*E(aX) =a*E(X)*
E(X+a)= E(X) + a
E(X+Y)= E(X) + E(Y)

*E(XY) = E(X) * E(Y)* if X & Y are <u>independent</u>

Variance of a Random Variable

- a weighted average of *squared deviations from the mean*, $[x - E(x)]^2$

$$Var(X) = E[(X-\mu)^2] = \sigma^2$$

  - $Var(X) = \sum_{x=-\infty}^{\infty} \left(x - E(x)\right)^2 p(x)$ for a *discrete* RV
  - $Var(X) = \int_{x=-\infty}^{\infty} \left(x - E(x)\right)^2 f(x)\,dx$ for a *continuous* RV

$$Var(X) = E[(X-\mu)^2] \;\Rightarrow\; E(X^2) - [E(X)]^2 \quad \text{for } \textbf{\textit{any}} \text{ RV}$$

Example:
Y = # heads in two tosses of a fair coin

P(Y = 0) = ¼
P(Y = 1) = ½
P(Y = 2) = ¼ and zero otherwise

$$\mu = E(Y) = 1$$

$$Var(Y) = (0-\mu)^2 \bullet P(Y=0) + (1-\mu)^2 \bullet P(Y=1) + (2-\mu)^2 \bullet P(Y=2)$$
$$= (0-1)^2 \bullet \frac{1}{4} \quad + (1-1)^2 \bullet \frac{1}{2} \quad + (2-1)^2 \bullet \frac{1}{4}$$
$$= \frac{1}{2}$$

OR

$$E(Y^2) = 0^2 \bullet P(Y=0) + 1^2 \bullet P(Y=1) + 2^2 \bullet P(Y=2)$$
$$= 0 \bullet \frac{1}{4} + 1 \bullet \frac{1}{2} + 4 \bullet \frac{1}{4} = \frac{3}{2}$$
$$Var(Y) = E(Y^2) - [E(Y)]^2 = \frac{3}{2} - 1^2 = \frac{1}{2}$$

Properties of Variance

Var(X±a) = *Var(X)*
Var(aX) = a^2***Var(X)*

Var(X+Y) = *Var(X) + Var(Y)* if X & Y are independent

Var(X+Y) = *Var(X) + Var(Y) + 2*Cov(X,Y)* always

*Cov(X,Y) = E([X–E(X)] * [Y–E(Y)])*
*= E(XY) – E(X)*E(Y)*

$$Y_1, Y_2, \ldots, Y_n \overset{iid}{\sim} N(\mu, \sigma^2)$$

→ a *Simple Random Sample* (SRS) of size *n*

i.i.d. → independent and identically distributed

$$E(\bar{Y}) = E\left(\frac{Y_1 + Y_2 + \ldots + Y_n}{n}\right) = \frac{1}{n} E\left(Y_1 + Y_2 + \ldots + Y_n\right) = \frac{1}{n}(n \bullet \mu) = \mu$$

$$Var(\bar{Y}) = Var\left(\frac{Y_1 + Y_2 + \ldots + Y_n}{n}\right) = \frac{1}{n^2} Var\left(Y_1 + Y_2 + \ldots + Y_n\right) = \frac{1}{n^2}\left(n \bullet Var(Y_i)\right) = \frac{\sigma^2}{n}$$

$$\frac{\bar{Y} - \mu}{\sigma / \sqrt{n}} = Z \sim N(0,1)$$

2-Sample Tests

$$Y_{11}, Y_{12}, \ldots, Y_{1n_1} \overset{iid}{\sim} N(\mu_1, \sigma_1^2)$$

$$H_0: \mu_1 = \mu_2$$

$$Y_{21}, Y_{22}, \ldots, Y_{2n_2} \overset{iid}{\sim} N(\mu_2, \sigma_2^2)$$

$$E(\bar{Y}_1 - \bar{Y}_2) = E(\bar{Y}_1) - E(\bar{Y}_2) = \mu_1 - \mu_2$$

$$Var(\bar{Y}_1 - \bar{Y}_2) = Var(\bar{Y}_1) + Var(\bar{Y}_2) = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$$