A 3-Dimensional Model of a “Toy Climate” Convection Loop

Natural convection in the atmosphere is fundamental to the weather patterns we observe every day. The process can be isolated for study by filling a hula-hoop shaped loop with fluid, and then heating (cooling) the bottom (top) half to create a temperature inversion (Left) [1]. A high-dimensional CFD simulation of such a loop was generated to represent a “toy climate”, for which we attempt to make forecasts with a 3-dimensional model.

Learning from the Past: Empirical Correction of the Forecast Model

To test the effectiveness of empirical correction, 1000 trial forecasts were performed starting from randomly selected, independent initial states after the training period. (Below) One particularly positive outcome. Trajectories of the true system (black), uncorrected model (blue) and corrected model (red) all starting from the same initial state (black circle). The corrected model remains close to the truth for far longer than the uncorrected model. (Right) Average error statistics over the 1000 trials. (Top) A forecast is generally considered useful for as long as its anomaly correlation (AC) remains above 0.6 [5]. Empirical correction increases the average duration of usefulness by approximately 30%. (Bottom) Average error in predicted fluid velocity $\Delta u$, taken relative to the natural variability of the system.

Incorporating System-Specific Dynamical Knowledge into the Correction Procedure

The general empirical correction procedure can be tailored to take advantage of system-specific dynamical knowledge. For this example, it is known to have two distinct regimes characterized by opposite directions of flow in the tube, (corresponding to the left and right lobes in state-space). (Right) Computing a different bias term $b$ and operator $L$ for each regime, and applying them appropriately in a forecast scenario, results in further error reduction. Results are also shown for corrected models incorporating knowledge of distance from equilibrium (from the centers of the lobes), and models incorporating both types of dynamical knowledge. Another reason to adapt the correction procedure: (Below) A corrected model gone wrong. Empirical correction has altered the stability of the convective equilibria, which now attract nearby states in contrast with the true system. See (Left) for a discussion of this unfortunate consequence.

Improved error statistics hide some important dynamical consequences of empirical correction, including broken symmetry and altered stability of equilibrium solutions. (Above) Difference (in minutes) between predicted and actual time of first regime change (flow-reversal), taken $E_{\text{corrected}} - E_{\text{uncorrected}}$ and plotted by initial state, for the (A) corrected, (B) lobe-dependent corrected, (C) equilibrium-dependent, and (D) lobe and equilibrium-dependent corrected models. The green dots show good predictions, while the red dots show initial states in the basins of the spuriously stable convective equilibria, (inset histograms show frequency of each). The lobe and equilibrium dependent corrected model is the best dynamical match to the truth.

References and Acknowledgements

The authors would like to acknowledge support from NSF EPSCoR, NASA EPSCoR, the Vermont Space Grant Consortium, and an NSF grant for the Mathematics & Climate Research Network.