Extremal Graph Thoery Homework 11

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Note, s is included in the following problems for completeness. Feel free to work independently of s or let $s = 2$, whatever feels most convenient.

1. Consider $T(6, 4)$ (fig 1)
   - (a) Calculate $\text{gn}(T(6, 4), s)$
   - (b) Let $\text{gn}(T(6, 4), s) = k$. Is $T(6, 4)$ saturated with respect to having guessing number at least $k + \epsilon$ for some ϵ? (that is, if we add an edge, does the guessing number necessarily increase)?
2. Recall what we know about $T(n, 4)$ (See Diestel Chapter 7 for a refresher on the structure of Turán graphs) and the following facts about guessing numbers:
   - $\text{gn}(K_n, s) = n - 1$
   - $\text{gn}(G, s) \geq n - \text{cp}(G)$
   - $\text{gn}(G, s) \leq n - \alpha(G)$
   - If G has disjoint subgraphs H_1 and H_2, $\text{gn}(G, s) \geq \text{gn}(H_1, s) + \text{gn}(H_2, s)$

   - (a) Bound $\text{gn}(T(n, 4), s)$ from above
   - (b) With your bound and the above facts about the guessing number, can you determine if $T(n, 4)$ is saturated with respect to some guessing number?
3. With the above facts about the guessing number, can you determine if $T(n, r)$ is saturated with respect to some guessing number?
4. Pick a favorite graph invariant
   - (a) Can you construct an example of a graph that's "saturated" with respect to that invariant?
   - (b) Do you think being saturated with respect to that invariant is related to local substructure in some way (think about a possible forbidden family of subgraphs)?

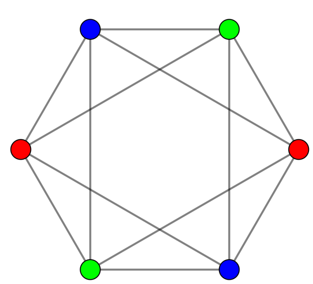

Figure 1: $T(6, 4)$ - Diagram generated by `https://commons.wikimedia.org/wiki/User:Tomruen`