Exercise 1 (Diestel 7.2)

Given $k > 0$, determine the extremal graphs of chromatic number at most k.

. . . . . .

Exercise 2 (Diestel 7.6)

(Prove Mantel's theorem, without using Turán's theorem)

Without using Turán's theorem, show that the maximum number of edges in a triangle-free graph of order $n > 1$ is $\lfloor n^2/4 \rfloor$.

. . . . . .

Exercise 3 (Diestel 7.7)

Show that

$$t_{r-1} \leq \frac{1}{2} n^2 \frac{r-2}{r-1},$$

with equality whenever $r - 1$ divides n.

. . . . . .

Exercise 4 (Diestel 7.8)

Show that $t_{r-1}/\binom{n}{2}$ converges to $(r-2)/(r-1)$ as $n \to \infty$.

Note: $t_{r-1}(n)/\binom{n}{2}$ is the *edge density* of Turán graph $T(n, r)$ (which is $(r-1)$-partite on $n \geq r - 1$ vertices, $t_{r-1}(n)$ is the number of edges in $T(n, r)$).

. . . . . .