Extremal Graph Theory - Graph Saturation - HW 7

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1. Show that $\text{sat}(P_5, 8) = 7$
2. Show that $\text{sat}(K_3, n) = n - 1$
3. Explain why a $(d-1)$-regular graph is $K_{1,d}$-saturated. Is it optimal?
4. Find the Kászonyi-Tuza bound for complete graphs and stars, and compare your results to the precise saturation number for both.
5. Recall ℓ and d from the Kászonyi-Tuza upper bound. Is a $K_{1,d-1}$-saturated graph necessarily $\mathcal{F}^{(\ell)}$-saturated? If so, why? If not, give a counter example.
6. Conjecture (Kászonyi-Tuza): For every graph H, $\lim_{n\to\infty} \frac{\text{sat}(n,H)}{n} = c < \infty$.
   (a) Kászonyi and Tuza showed that $\text{sat}(H, n) < cn$ for some c. Why is this conjecture not a corollary of that result?
   (b) Convince yourself the conjecture is true for complete graphs and stars.
   (c) Can you find any graphs where $\lim_{n\to\infty} \frac{\text{sat}(H,n)}{n} = \frac{wt(H)-1}{2}$? These are called Sat-sharp graphs, term coined by Cameron and Puleo.