Exercise 1 (5.8)

Let $n = |G|$ and $\Delta = \Delta(G)$. Prove that there is a partition $V(G) = \bigcup_{i=1}^{k} V_i$ with $k \leq \lfloor \Delta/2 \rfloor + 1$, such that $\Delta(G[V_i]) \leq 1$ for every i. In other words, each $G[V_i]$ consists of independent edges and isolated vertices.

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Exercise 2 (5.22)

Let G be a critical k-chromatic graph. Prove by induction on k that any two vertices of G are joined by a path of length at least k.

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Exercise 3 (5.28)

Let G be a graph whose vertices are certain points of the plane and in which two vertices are joined if and only if they are at distance 1. (Note that this graph is far from planar; this has nothing to do with embeddings on surfaces.) Find upper or lower bounds on the chromatic number of this graph. This problem has seen a lot of progress since the book was written; it is called the Hadwiger-Nelson problem.

Is it true that if G does not contain a triangle then it is 3-colorable?

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