

Exercise 1

Let $m(n, k)$ be the minimum number of edges in a graph such that every pair of vertices is joined by at least k paths of length ≤ 2 . Let $1 < c < \frac{1}{2}(3 + \sqrt{5})$ and $n = \lfloor ck \rfloor$. Show that

$$m(n, k) \geq c^{3/2}k^2/2 + O(k).$$

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Exercise 2

Let $m_3(n, k)$ be described as above, but with paths of length ≤ 3 . Can you find any nontrivial bounds on this value?

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Exercise 3 (4.5)

Show that if G is connected, then

$$\chi(G) \leq |G| - \text{diam } G + 1,$$

where $\chi(G)$ is the chromatic number of G .

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