Exercise 1

Write a general construction for a graph that has exactly k 1-factors. Can you optimize in terms of number of vertices/connectivity?

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Exercise 2 (2.31)

Prove that a 2k-regular graph is the union of edge disjoint 2-factors.

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Exercise 3 (2.32)

Prove that the Petersen graph is not a union of edge-disjoint 1-factors.

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Exercise 4 (2.11)

Let G be an abelian group of order n. Let a_1, \ldots, a_n be a sequence of elements of G, possibly with repetitions. Prove that there exist two permutations g_1, \ldots, g_n and g'_1, \ldots, g'_n of G such that $a_i = g_i + G'_i$, $1 \le i \le n$, if and only if $\sum_{i=1}^n a_i = 0$.

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Exercise 5 (2.36)

(Conjecture.) every 4-regular graph contains a 3-regular subgraph.

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