GENERALIZED EXTREMAL EXERCISES

Recall, $ex(n, H, F)$ denotes the maximum number of copies of H in an n-vertex, F-free graph. $\mathcal{N}(H, G)$ denotes the number of copies of H contained in a graph G.

1. Recall, $ex(n, K_r, K_k) = \mathcal{N}(K_r, T_{k-1}(n))$, where $T_{k-1}(n)$ is the Turán graph on n vertices with $k - 1$ parts. Like Turán's Theorem, this result has been rediscovered many times, and multiple proofs are possible. Reprove the result without using Zykov symmetrization.

2. Let C_k determine the cycle on k edges, and P_k the path on k edges. (So P_k is not a subgraph of C_k). Asymptotically determine $ex(n, C_k, P_k)$.

3. Prove that for any pair of graphs H, F where H has h vertices, the function $\frac{ex(n,H,F)}{\binom{n}{h}}$ is monotone decreasing. (Hint: Let G be an F-free, n-vertex graph with the maximum number of copies of H, and double-count the pair (H, v) where H is a copy of the graph H in G and v is a vertex not in H.)