251 Abstract Algebra - Midterm 2 Practice

Name:

Justify all of your answers.

Question 1

Let H be a subgroup of G and fix some element $g \in G$.

(a) Prove that gHg^{-1} is a subgroup of G. [4 points]

(b) Prove that $|gHg^{-1}| = |H|$. [3 points]

(c) Describe the subgroup $s\langle r \rangle s^{-1}$ of D_8. [3 points]

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Question 2

Prove that if H and K are both normal subgroups of G then their intersection $H \cap K$ is also a normal subgroup. [10 points]

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Question 3

Let H and K be subgroups of G. Draw all possible lattices on the set $G, 1, H, K, H \cap K, \langle H, K \rangle$. [10 points]

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Question 4

Consider the subgroup H of S_5 generated by (1 2) and (1 6).

(a) What is the order of H? [5 points]

(b) Is H normal in S_5? [5 points]

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Question 5

Let G be a group and suppose that $gNg^{-1} \subseteq N$ for all $g \in G$.

(a) Find a homomorphism $\phi : G \to G/N$ such that N is the kernel of ϕ. [6 points]

(b) Show that the left and right cosets of N induce the same partition of G. [4 points]

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