251 Abstract Algebra - Midterm 2

Name:

Justify all of your answers.

This exam has 5 questions. Your score will be determined by your 3 best questions. (Max score: 30.)

Question 1

Let $\phi : G \to H$ be a homomorphism and let E be a subgroup of H.

(a) Prove that $\phi^{-1}(E) \leq G$. [7 points]

(b) Let $N = \langle r^2 \rangle$ be a subgroup of D_8. You may assume $N \trianglelefteq D_8$. Consider the projection homomorphism $\phi : D_8 \to D_8/N$, which maps g to gN for each $g \in D_8$. What is $\phi^{-1}(\langle sN \rangle)$? [3 points]

....

Question 2

List all subgroups of S_3 and draw the subgroup lattice for S_3. Choose one non-trivial subgroup and give its normalizer. [10 points]

....

Question 3

Prove that if H is a subgroup of G, then $\langle H \rangle = H$. [10 points]

....

Question 4

Lagrange's Theorem states that if G is a finite group and $H \leq G$, then the order of H divides the order of G. You may assume that the left cosets of H form a partition of G. From here, finish the proof of Lagrange's Theorem (i.e. show that all left cosets have the same number of elements). [10 points]

....

Question 5

(a) Show that if G is an abelian group, every subgroup N of G is normal. [6 points]

(b) Show that for any G (not necessarily abelian) if $N \leq Z(G)$ then N is normal. [4 points]

....