251 Abstract Algebra - Midterm 1a

Name:

Justify all of your answers.

Question 1

Let $\sigma \in S_8$ be the following permutation:

$$\begin{array}{ll} 1 \mapsto 3 & 5 \mapsto 2 \\ 2 \mapsto 4 & 6 \mapsto 6 \\ 3 \mapsto 1 & 7 \mapsto 7 \\ 4 \mapsto 5 & 8 \mapsto 8. \end{array}$$

(a) Find the cycle decomposition of σ and σ^{-1} and write it in the standard format. [4 points]

(b) Find $|\sigma|$. [3 points]

(c) Find $(6\ 8\ 7)\sigma(6\ 7\ 8)$. [3 points]

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Question 2

Prove that for a group G with $|G| = n > 2$ it is not possible to have a subgroup H with $|H| = n - 1$. [10 points]

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Question 3

For a group G and subset $A \subseteq G$, let $N_G(A)$ be the normalizer of A in G and $C_G(A)$ the centralizer of A in G. Show that $C_G(A) \leq N_G(A)$ and $Z(G) \leq N_G(A)$. [10 points]

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Question 4

Consider the dihedral group D_{2n}:

$$D_{2n} = \langle r, s \mid r^n = s^2 = 1,\ rs = sr^{-1} \rangle.$$

Show that we can also write [10 points]

$$D_{2n} = \langle a, b \mid a^2 = b^2 = (ab)^n = 1,\ rs = sr^{-1} \rangle,$$

by letting $a = s$ and $b = sr$.

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Question 5

(a) For a group G acting on a set S. Let G_s be the stabilizer of $s \in S$ of the action. Show that $g \in G_s$ implies that $g^{-1} \in G_s$. (This is part of the proof of showing that the stabilizer is a subgroup of G.) [5 points]

(b) Let H be a subgroup of order 2 in G. Show that $N_G(H) = C_G(H)$. [5 points]

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