251 Abstract Algebra - Midterm 1 - Practice

Name:

Justify all of your answers.

Question 1

Let $\sigma \in S_6$ be the following permutation:

$$\begin{array}{ll} 1 \mapsto 3 & 4 \mapsto 1 \\ 2 \mapsto 4 & 5 \mapsto 6 \\ 3 \mapsto 2 & 6 \mapsto 5. \end{array}$$

(a) Find the cycle decomposition of σ and σ^{-1}.

(b) Find $|\sigma|$.

(c) Consider the element $\tau = (1\ 2\ 3)$. Find two elements $\tau_1, \tau_2 \in S_6$ such that $|\tau_1| = |\tau_2| = 2$ and $\tau = \tau_1 \tau_2$.

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Question 2

Let H be a nonempty subset of a group G, and suppose that for all $x, y \in H$, we have $xy^{-1} \in H$. Show that for all $x \in H$, we have $x^{-1} \in H$. (This is part of the proof of the Subgroup Criterion.)

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Question 3

For a group G and subset $A \subseteq G$, let $C_G(A)$ be the centralizer of A in G, and let $Z(G)$ be the center. Show that $C_G(Z(G)) = G$.

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Question 4

Find an injective homomorphism $\phi : C_3 \to S_4$, by giving an explicit injective map and showing that it is indeed a homomorphism.

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Question 5

(a) For a group G acting on a set S. Let G_s be the stabilizer of $s \in S$ of the action. Show that G_s is closed under multiplication. (This is part of the proof of showing that the stabilizer is a subgroup of G.)

(b) Let D_8 act on the corners of a square in the usual way. Number the corners in clockwise order as $\{1, 2, 3, 4\}$. Then $\sigma_r = (1\ 2\ 3\ 4)$ and $\sigma_s = (1\ 2)(3\ 4)$. Find $(D_8)_1$, i.e. the stabilizer of 1 in D_8.

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