251 Abstract Algebra - Midterm 1

Name:

Justify all of your answers.

Question 1

Let $\sigma \in S_8$ be the following permutation:

$$\begin{array}{ll} 1 \mapsto 3 & 5 \mapsto 2 \\ 2 \mapsto 4 & 6 \mapsto 6 \\ 3 \mapsto 8 & 7 \mapsto 7 \\ 4 \mapsto 5 & 8 \mapsto 1. \end{array}$$

(a) Find the cycle decomposition of σ and σ^{-1}. [4 points]

(b) Find $|\sigma|$. [3 points]

(c) Write σ as a product of (not necessarily disjoint) cycles of length 2. [3 points]

....

Question 2

Let H be a nonempty subset of a finite group G, and suppose that for all $x, y \in H$, we have $xy \in H$. Show that for all $x \in H$, we have $x^{-1} \in H$. (This is part of the proof of the Subgroup Criterion.) [10 points]

....

Question 3

For a group G and subset $A \subseteq G$, let $N_G(A)$ be the normalizer of A in G. Show that $N_G(A) \leq G$. [10 points]

....

Question 4

Consider the dihedral group D_{2n} where $n = 2k$ is an even number:

$$D_{2n} = \langle r, s \mid r^n = s^2 = 1,\ rs = sr^{-1} \rangle.$$

(a) Show that the element $z = r^k$ commutes with all elements of D_{2n}. [5 points]

(b) Show that z is the only non-identity element that commutes with all elements of D_{2n}. [5 points]

....

Question 5

(a) For a group G acting on a set S. Let G_s be the stabilizer of $s \in S$ of the action. Show that $g \in G_s$ implies that $g^{-1} \in G_s$. (This is part of the proof of showing that the stabilizer is a subgroup of G.) [6 points]

(b) Let a group G act on itself by conjugation: let $g \cdot h = ghg^{-1}$ for all $g, h \in G$. For a given element $a \in G$, describe the stabilizer G_a in terms of normalizers/centralizers/center. [4 points]

....

Question 6

Find a set of generators and relations for S_4. **[0 points]**

·········