251 Abstract Algebra - Final - Practice

Name:

The final will cover all material from this semester. This practice exam covers only material that was not on Midterm 1 or 2.

Justify all of your answers.

Question 1

Prove that σ^2 is an even permutation for every $\sigma \in S_n$.

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Question 2

Use the class equation to find all finite groups which have exactly two conjugacy classes.

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Question 3

Prove that if $P \in Syl_p(G)$ and H is a subgroup of G containing P, then $P \in Syl_p(H)$.

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Question 4

Show that the center of a direct product is the direct product of the centers:

$$Z(G_1 \times G_2 \times \cdots \times G_n) = Z(G_1) \times Z(G_2) \times \cdots \times Z(G_n).$$

Deduce that a direct product of groups is abelian if and only if each of the factors is abelian.

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Question 5

In each part, give the list of invariant factors for all abelian groups of the specified order:

(a) 270,

(b) 9801.

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