Classroom Exercises

Exercise 1  Triangle Inequality
Show that if $a$ and $b$ are any two real numbers, then

$$|a + b| \leq |a| + |b|.$$  

Exercise 2  Corollary
Show that if $a$ and $b$ are any two real numbers, then

$$|a - b| \geq ||a| - |b||.$$  

Exercise 3  Irrational numbers
Show that $\sqrt{2}$ is irrational.  

Exercise 4  Well-ordering
Prove that every non-empty set of positive integers contains a smallest member. This is called the well-ordering principle. (Ex. 1.6)

Exercise 5  Primes
Prove that there is no largest prime. (Ex. 1.1)

Homework Assignment
Write a friendly introduction to Euclid’s proof that there are infinitely many primes. The audience should be first-year undergraduate math majors. Explain how, given a set of primes $p_1, \ldots, p_r$, we can use Euclid’s method to find a new prime $p_{r+1}$. Suppose that we only know about the primes 2 and 7. Use your method and Sage to find at least five additional primes.