124 Linear Algebra - Midterm 2 - Practice

Name:

Justify all of your answers.

Question 1

Consider the map $h : \mathcal{P}_2 \to \mathcal{P}_2$ given by $h(a + bx + cx^2) = bx^2 - (a + c)x + a$.

(a) Show that this map is a homomorphism.

(b) Find the range space and the null space of this map.

(c) Let $B = \langle 1, x, x + x^2 \rangle$. Find $Rep_B(1 + x^2)$ and $Rep_B(h(1 + x^2))$.

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Question 2

Let $f : V \to W$ be a homomorphism. Show that f preserves linear dependence. In other words, if a set $(\vec{v}_1, \ldots, \vec{v}_k)$ is linearly dependent in V, then $(f(\vec{v}_1), \ldots, f(\vec{v}_k))$ is linearly dependent in W.

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Question 3

Let $B = \langle\vec{\beta}_1, \vec{\beta}_2, \vec{\beta}_3, \vec{\beta}_4\rangle$ be a basis for a vector space V. Find a matrix with respect to B, B for each of the following transformations of V determined by:

(a) $\vec{\beta}_1 \mapsto \vec{\beta}_2$, $\vec{\beta}_2 \mapsto \vec{\beta}_3$, $\vec{\beta}_3 \mapsto \vec{\beta}_4, \vec{\beta}_4 \mapsto \vec{0}$.

(b) $\vec{\beta}_1 \mapsto \vec{\beta}_2$, $\vec{\beta}_2 \mapsto \vec{0}$, $\vec{\beta}_3 \mapsto \vec{\beta}_4, \vec{\beta}_4 \mapsto \vec{\beta}_1$.

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Question 4

Consider the following matrix

$$A = \begin{pmatrix} 1 & 1 & -3 \\ 2 & 5 & 0 \end{pmatrix}.$$

(a) Find the dimension of the domain and the codomain of an map represented by this matrix.

(b) Give an explicit example of a domain with appropriate basis, a codomain with appropriate basis, and a map such that the matrix A represents the map with respect to your chosen bases.

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Question 5

Let V, W, U be vector spaces and consider two homomorphisms $f : V \to W$ and $h : W \to U$.

(a) Use the rank nullity theorem to show that the rank of $h \circ f$ is at most the minimum of the ranks of h and f.

(b) To show that the rank of $h \circ f$ can be less than that, give an example of two 2×2 matrices A, B that both have rank 1, such that AB has rank 0.

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