124 Linear Algebra - Midterm 2

Name:

Justify all of your answers.

This exam has 5 questions. Your score will be determined by your 4 best questions. (Max score: 40.)

Question 1

Consider the map $h : \mathcal{P}_2 \to \mathcal{P}_2$ given by $h(a + bx + cx^2) = (a + b) + (a + b)x + cx^2$.

(a) Show that this map is a homomorphism.

(b) Find the range space and the null space of this map.

(c) Let $B = \langle 1, x, x + x^2 \rangle$. Find $Rep_B(1 + x + x^2)$ and $Rep_B(h(1 + x + x^2))$.

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Question 2

Assume h is a linear transformation of V and that $\langle \vec{\beta}_1, \ldots, \vec{\beta}_n \rangle$ is a basis of V. Prove the following statements.

(a) If $h(\vec{\beta}_i) = \vec{0}$ for each basis vector then h is the zero map.

(b) If $h(\vec{\beta}_i) = \vec{\beta}_i$ for each basis vector then h is the identity map.

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Question 3

Consider reflection through the x-axis in $\mathbb{R}^2$. This is the map $f : \mathbb{R}^2 \to \mathbb{R}^2$ that sends $\binom{x}{y}$ to $\binom{x}{-y}$.

(a) Find the matrix that represents this map, with respect to the standard basis $\langle \binom{1}{0}, \binom{0}{1} \rangle$. [5 points]

(b) Find the matrix that represents this map, with respect to the basis $B = \langle \binom{1}{1}, \binom{1}{0} \rangle$. [5 points]

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Question 4

Consider the following matrix

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

(a) Find the dimension of the null space and the range space of a map represented by this matrix. [5 points]

(b) Suppose that this matrix represents a linear map $f : \mathcal{P}_2 \to \mathbb{R}^2$ with respect to their standard bases. What is $f(x)$? What is $f^{-1}(\left(\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}\right))$? [5 points]

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Question 5

Let V, W be vector spaces and consider a homomorphism $f : V \to W$.

(a) Show that the null space $\mathcal{N}(f)$ is a subspace of V. [5 points]

(b) Use the rank nullity theorem to show that the rank of f is at most the dimension of V. [5 points]

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