124 Linear Algebra - Midterm 1a

Name:

Justify all of your answers.

Question 1

Let

$$S = \left\{ \begin{pmatrix} a+b \\ b+c \\ a+c \\ a+b+c \end{pmatrix} \mid a, b, c \in \mathbb{R} \right\}.$$

(a) Show that S is closed under taking linear combinations of two vectors. [3 points]

(b) You may assume that S is a subspace. Find a basis B for S. [4 points]

(c) Use your basis B to write the vector $\vec{v} = \begin{pmatrix} 2 \\ 3 \\ 3 \\ 4 \end{pmatrix} \in S$ as represented by B, i.e. find the vector [3 points]

$$\mathrm{Rep}_B(\vec{v}).$$

Hint: your answer should be a vector of length 3, but the entries will depend on your choice of basis in (b).

....

Question 2

Consider the following subset of $\mathcal{P}_4$:

$$S = \{p(x) \in \mathcal{P}_4 \mid p(0) = 0\}.$$

(a) Show that S is a subspace of $\mathcal{P}_4$. [5 points]

(b) Find a basis B for S. (Prove that your proposed set B is indeed a basis.) [5 points]

• • • • • • • • •

Question 3

Consider three vectors $\vec{u}, \vec{v}, \vec{w}$ in a vector space V.

(a) In general, is the vector $\vec{u}$ in the span of the set $(\vec{u} + \vec{v}, \vec{w})$? Prove or give a counterexample. [5 points]

(b) Suppose that the set $(\vec{u}, \vec{v}, \vec{w})$ is linearly independent. What is the dimension of the subspace $[(\vec{u} + \vec{v}, \vec{v} + \vec{w}, \vec{u} + \vec{w})]$? [5 points]

....

Question 4

Suppose that W is a subspace of $\mathbb{R}^3$ and $W \neq \mathbb{R}^3$.

(a) Show that $\dim W \leq 2$. [6 points]

(b) Suppose that $\dim W = 2$, and $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \in W$ and $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \notin W$. Does this information fully define the subspace W? [4 points]

....

Question 5

We would like to solve the following system of equations:

$$\begin{array}{rrrcl} 3x & +6y & & = & 3 \\ x & +2y & +z & = & 3 \\ x & +2y & & = & 2. \end{array}$$

(a) Find an explicit matrix A and vector $\vec{w}$ and express the system of equations in the form [2 points]

$$A\vec{x} = \vec{w},$$

where $\vec{x} = \left(\begin{smallmatrix} x \\ y \\ z \end{smallmatrix}\right)$.

(b) Find the augmented matrix of this system, and then write your augmented matrix in RREF. [4 points]

(c) What is the set of solutions to this system? [2 points]

(d) What is the rank of A? [2 points]

....

Question 6

A library has n books and $n + 1$ subscribers. Each subscriber read at least one book from the library. **[0 points]** Prove that there must exist two disjoint sets of subscribers who read exactly the same books (that is, the union of the books read by the subscribers in each set is the same).

• • • • • • • • •