124 Linear Algebra - Midterm 1 - Practice

Name:

Justify all of your answers.

Question 1

Let

$$S = \left\{ \begin{pmatrix} a \\ b \\ c \\ a+b+c \end{pmatrix} \mid a, b, c \in \mathbb{R} \right\}.$$

(a) Show that S is closed under scalar multiplication.

(b) You may assume that S is a subspace. Find a basis B for S.

(c) Use your basis B to write the vector $\vec{v} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 6 \end{pmatrix} \in S$ as represented by B, i.e. find the vector

$$\mathrm{Rep}_B(\vec{v}).$$

Hint: your answer should be a vector of length 3, but the entries will depend on your choice of basis in (b).

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Question 2

Let V be a vector space. Show (carefully and concisely) that V has exactly one subspace on a finite number of elements.

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Question 3

Consider a pair of vectors $\vec{u}, \vec{v}$ in a vector space V. We will show that

$$[(\vec{u}, \vec{v})] = [\vec{u} + \vec{v}, \vec{u} - \vec{v}],$$

by showing that $[(\vec{u}, \vec{v})] \subseteq [\vec{u} + \vec{v}, \vec{u} - \vec{v}]$ and then that $[\vec{u} + \vec{v}, \vec{u} - \vec{v}] \subseteq [(\vec{u}, \vec{v})]$.

(a) Show that $[(\vec{u}, \vec{v})] \subseteq [\vec{u} + \vec{v}, \vec{u} - \vec{v}]$. Note that spans are subspaces, which are therefore closed under taking linear combinations. Therefore, to show that $[(\vec{u}, \vec{v})] \subseteq W$, you only need to show $\vec{u}, \vec{v} \in W$.

(b) Show that $[\vec{u} + \vec{v}, \vec{u} - \vec{v}] \subseteq [(\vec{u}, \vec{v})]$.

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Question 4

Find a basis B for the vector space

$$\mathcal{P}_2 = \{a_0 + a_1 x + a_2 x^2 \mid a_0, a_1, a_2 \in \mathbb{R}\}$$

such that your basis includes the element $1+x$. You may assume that $\dim \mathcal{P}_2$=3. Therefore, it is sufficient to find a set of 3 elements and show that it is a linearly independent set.

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Question 5

We would like to solve the following system of equations:

$$\begin{array}{rrrcr} 2x & & & = & 6 \\ 2x & +y & +z & = & 5 \\ 4x & +y & +z & = & 11. \end{array}$$

(a) Find an explicit matrix A and vector $\vec{w}$ and express the system of equations in the form

$$A\vec{x} = \vec{w},$$

where $\vec{x} = \left(\begin{smallmatrix} x \\ y \\ z \end{smallmatrix}\right)$.

(b) Find the augmented matrix of this system, and then write your augmented matrix in RREF.

(c) What is the set of solutions to this system?

(d) What is the rank of A?

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