124 Linear Algebra - Final - Practice

Name:

The final will cover the material from the entire semester. This practice exam only covers material not on Midterm 1 or 2.

Justify all of your answers.

Question 1

(a) Without justification, cross out the items in the list which do not apply:

For **every** square matrix A, if we know the characteristic polynomial of a A, then we can infer the

- eigenvalues
- eigenvectors
- eigenspaces
- trace
- determinant
- algebraic multiplicities
- geometric multiplicities
- invertibility
- rank

of A.

(b) *[6 points]* Find all the eigenvalues and eigenspaces of

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 4 & 4 & 2 \\ 8 & 0 & 5 \end{pmatrix}.$$

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Question 2

Let $L \subset \mathbb{R}^2$ be the span of $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$.

(a) *[4 points]* Give the 2×2 matrix A of the projection onto L, and find the projections of the standard basis vectors. *Hint: use Gram-Schmidt.*

(b) *[6 points]* Is A diagonalizable? If so, give an invertible matrix S and a diagonal matrix B, such that $A = SBS^{-1}$. Otherwise, explain why A is not diagonalizable.

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Question 3

(a) *[6 points]* Diagonalize the following matrix:

$$A = \begin{pmatrix} 5 & 4 \\ 0 & 1 \end{pmatrix}$$

(b) *[4 points]* Can you diagonalize A^2? In general, what do the powers $A^2, A^3, \ldots$ of a diagonalizable matrix look like? *Hint: do not try to square A directly. Instead notice that* $SBS^{-1}SBS^{-1} = SB^2S^{-1}$.

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Question 4

(a) *[6 points]* Show that the composition of lineaar transformations on $\mathbb{R}^1$ is commutative. *Hint: you have shown before that linear transformations on $\mathbb{R}^1$ have a very restricted form.*

(b) *[4 points]* Is this true for linear transformations on $\mathbb{R}^n$ in general? Prove or give a counter-example.

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Question 5

Show that λ is an eigenvalue of an $n \times n$ matrix A if and only if the map represented by $A - \lambda I_n$ is not an isomorphism. *Hint: whaat do you know about the kernel of isomorphisms?*

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