Exercise 1 (3.23 p. 421)

For each, find the characteristic polynomial and the eigenvalues.

(a) $\begin{pmatrix} 10 & -9 \\ 4 & -2 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$

(c) $\begin{pmatrix} 0 & 3 \\ 7 & 0 \end{pmatrix}$

(d) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

(e) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

. . . .

Exercise 2

We define the **trace** of a matrix as the sum of its diagonal entries: $\text{tr}(A) = a_{11} + a_{22} + \ldots + a_{nn}$. Show that for any 2×2 matrix A, we have

$$f_A(\lambda) = \lambda^2 - \text{tr}(A)\lambda + \det(A)$$

. . . .

Exercise 3

For which values of x does the following matrix have $\det(A) = \text{tr}(A) = 0$?

$$A = \begin{pmatrix} 1 & 7 & 9 \\ 0 & 1+x & 7 \\ 0 & 0 & x \end{pmatrix}.$$

. . . .

Exercise 4 (3.31 p. 422)

Find the eigenvalues and associated eigenvectors of the matrix representing the differentation operator $d/dx : \mathcal{P}_2 \to \mathcal{P}_2$.

. . . .

Exercise 5

Suppose that a 2×2 matrix A is such that $(A\vec{v}) \cdot \vec{v} = 0$, for all $\vec{v} \in \mathbb{R}^2$. Can A be invertible? What if A is 3×3, and $(A\vec{v}) \cdot \vec{v} = 0$, for all $\vec{v} \in \mathbb{R}^3$?

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