Carefully justify every answer.

Exercise 1

Show that for an $n \times p$ matrix A and a $p \times m$ matrix B, that if the $\text{im}(B) = \ker(A)$, then $\text{im}(AB) = \{\vec{0}\}$.

....

Exercise 2

Suppose that V is a subspace of $\mathbb{R}^n$. Prove that any linear transformation $T : V \to V$ can be extended to a linear transformation $S : \mathbb{R}^n \to \mathbb{R}^n$. In other words, prove that for any such $T(\vec{v})$ there exists an $S(\vec{x})$ (defined for all $\vec{x} \in \mathbb{R}^n$) such that $S(\vec{v}) = T(\vec{v})$ for all $\vec{v} \in V$.

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