Carefully justify every answer.

Exercise 1 (2.26 p.209)

In general, what is the null space of the differentation transformation $d/dx : \mathcal{P}_n \to \mathcal{P}_n$? What is the null space of the second derivative? The kth derivative?

. . . .

Exercise 2 (2.30 p.209)

For the map $f : \mathbb{R}^2 \to \mathbb{R}$ given by $f\left(\left(\begin{smallmatrix} x \\ y \end{smallmatrix}\right)\right) = 2x + y$ sketch the inverse image sets $f^{-1}(-3)$, $f^{-1}(0)$ and $f^{-1}(1)$.

. . . .

Exercise 3 (2.37 p.209)

Show that a linear map is one-to-one if and only if it preserves linear independence.

. . . .

Exercise 4 (1.17 p.220)

For a homomorphism from $\mathcal{P}_2$ to $\mathcal{P}_3$ that sends

$$1 \mapsto 1 + x,\ x \mapsto 1 + 2x,\ x^2 \mapsto x - x^3,$$

where does $1 - 3x + 2x^2$ go? Represent this linear map as a matrix with respect to the standard bases for $\mathcal{P}_2$ and $\mathcal{P}_3$.

. . . .

Exercise 5 (1.20 p.220)

Represent the homomorphism $h : \mathbb{R}^3 \to \mathbb{R}^2$ given by $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \mapsto \left(\begin{smallmatrix} x+y \\ x+z \end{smallmatrix}\right)$ with respect to these bases:

$$B = \left\langle \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right\rangle, \quad D = \left\langle \left(\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}\right), \left(\begin{smallmatrix} 0 \\ 2 \end{smallmatrix}\right) \right\rangle.$$

. . . .

Exercise 6 (1.26 p.221)

Consider the reflection map $f_l : \mathbb{R}^2 \to \mathbb{R}^2$ which reflects all vectors across a line l through the origin. Express this transformation as a matrix with respect to the standard basis. Note that we can define the line l by an angle $0 \leq \theta < \pi$.

. . . .