Carefully justify every answer.

Exercise 1 (1.19 p.196)

Are each of the following maps $f : \mathcal{M}_{2\times 2} \to \mathbb{R}$ linear? Justify your answers.

(a) $f\left(\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right)\right) = a + d$,

(b) $f\left(\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right)\right) = ad - bc$,

(c) $f\left(\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right)\right) = a + b + c + d + 1$,

. . . .

Exercise 2 (1.20 p.196)

We have seen that $f : \mathcal{P}_3 \to \mathcal{P}_2$ given by $f(p(x)) = \frac{dp(x)}{dx}$ is a linear map. What about the map $f : \mathcal{P}_2 \to \mathcal{P}_3$ given by the indefinite integral (where we set the constant to 0), i.e. $f(a + bx + cx^2) = ax + (b/2)x^2 + (c/3)x^3$?

. . . .

Exercise 3 (1.26 p.197)

Part of the definition of a linear function is that it respects addition. Does a linear function respect subtraction?

. . . .

Exercise 4 (1.27 p.197)

Assume h is a linear transformation of V and that $(\vec{\beta}_1, \ldots, \vec{\beta}_n)$ is a basis of V. Prove the following statements.

(a) If $h(\vec{\beta}_i) = \vec{0}$ for each basis vector then h is the zero map.

(b) If $h(\vec{\beta}_i) = \vec{\beta}_i$ then h is the identity map.

. . . .

Exercise 5 (1.32 p.198)

Show that every homomorphism from $\mathbb{R}^1$ to $\mathbb{R}^1$ acts via multiplication by a scalar.

. . . .

Exercise 6 (2.21 p.208)

Let $h : \mathcal{P}_3 \to \mathcal{P}_4$ be given by $p(x) \mapsto xp(x)$. Give the range space and null space.

. . . .