Carefully justify every answer.

Exercise 1

Consider three linearly independent vectors $\vec{v_1}, \vec{v_2}, \vec{v_3}$. Are the vectors $\vec{v_1}, \vec{v_1} + \vec{v_2}, \vec{v_1} + \vec{v_2} + \vec{v_3}$ also linearly independent?

. . . .

Exercise 2 (2.17)

Find a basis for, and the dimension of, each of the following spaces.

(a) the space

$$\left\{ \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \in \mathbb{R}^4 \mid x - w + z = 0 \right\},$$

(b) the set of 5×5 matrices whose only nonzero entries are on the diagonal,

(c) $\{a_0 + a_1 x + a_2 x^2 + a_3 x^3 \mid a_0 + a_1 = 0 \text{ and } a_2 - 2a_3 = 0\} \subseteq \mathcal{P}_3$.

. . . .

Exercise 3 (2.20)

Find the dimension of this subspace of $\mathbb{R}^2$:

$$S = \left\{ \begin{pmatrix} a+b \\ a+c \end{pmatrix} \mid a, b, c \in \mathbb{R} \right\}.$$

. . . .

Exercise 4 (2.37)

Assume U and W are both subspaces of some vector space, and that $U \subseteq W$.

(a) Prove that $\dim(U) \leq \dim(W)$.

(b) Prove that equality of dimension holds if and only if $U = W$.

. . . .

Exercise 5 (3.22)

Give a basis for the column space of this matrix. Give the matrix's rank:

$$\begin{pmatrix} 1 & 3 & -1 & 2 \\ 2 & 1 & 1 & 0 \\ 0 & 1 & 1 & 4 \end{pmatrix}.$$

. . . .

Exercise 6 (3.26)

Given $a, b, c \in \mathbb{R}$, what value of d will cause this matrix to have a rank of 1?

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

. . . .

Exercise 7

If A is a matrix in reduced row echelon form, and a column is removed, is it still in reduced row echelon form? What if a row is removed? Justify your answer carefully.

. . . .

Exercise 8

Read Section Three.IV and give an example of a 2×2 matrix and its inverse (that is not an example from the book).

. . . .