Carefully justify every answer.

## Exercise 1 (2.1.20)

Show that the set of $2 \times 2$ matrices with real entries under the usual matrix operations form a vector space.

.........

## Exercise 2 (2.1.22)

Show that the following set, under operations inherited from $\mathbb{R}^3$, is not a vector space:

$$\left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 = 1 \right\}.$$

.........

## Exercise 3 (2.1.24)

Is the set of rational numbers a vector space over $\mathbb{R}$ under the usual addition and scalar multiplication operations?

.........

## Exercise 4

Determine, in each case, whether $W$ is a subspace of $\mathbb{R}^3$. You may use Lemma 2.9 from page 98, or use only items (1), (4), (6) from Definition 1.1 on page 84 and assume that the rest are inherited from $\mathbb{R}^3$.

(a) $W = \left\{ \begin{pmatrix} a \\ a \\ a \end{pmatrix} \;\middle|\; a \in \mathbb{R} \right\}$,

(b) $W = \left\{ \begin{pmatrix} a+1 \\ a+2 \\ a+3 \end{pmatrix} \;\middle|\; a \in \mathbb{R} \right\}$,

(c) $W = \left\{ \begin{pmatrix} a \\ b \\ c \end{pmatrix} \;\middle|\; a, b, c \in \mathbb{R}, \quad a < b < c \right\}$,

.........

## Exercise 5

Can you find a subset of $\mathbb{R}^2$ that is closed under addition, but not scalar multiplication? Can you find a subset of $\mathbb{R}^2$ that is closed under scalar multiplication, but not addition?

.........

**Exercise 6**

For which values of $a$ and $b$ is $\begin{pmatrix} 2 \\ -4 \\ a \\ b \end{pmatrix}$ in the span of the set $\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 4 \\ 5 \end{pmatrix} \right\}$?

. . . . . . . . .

**Exercise 7**

Look through all of Sections Two.III.1-Two.III.3 (pages 121-143). You do not need to read every detail. Answer the following questions.

(a) What is the relationship between the row and column ranks of a matrix?

(b) Write down any terms you come across that you do not understand yet. (There is no wrong or right answer here. This is just to help me lecture.)

. . . . . . . . .