

Patterns of Vibrations

Natural Modes

Natural Frequencies



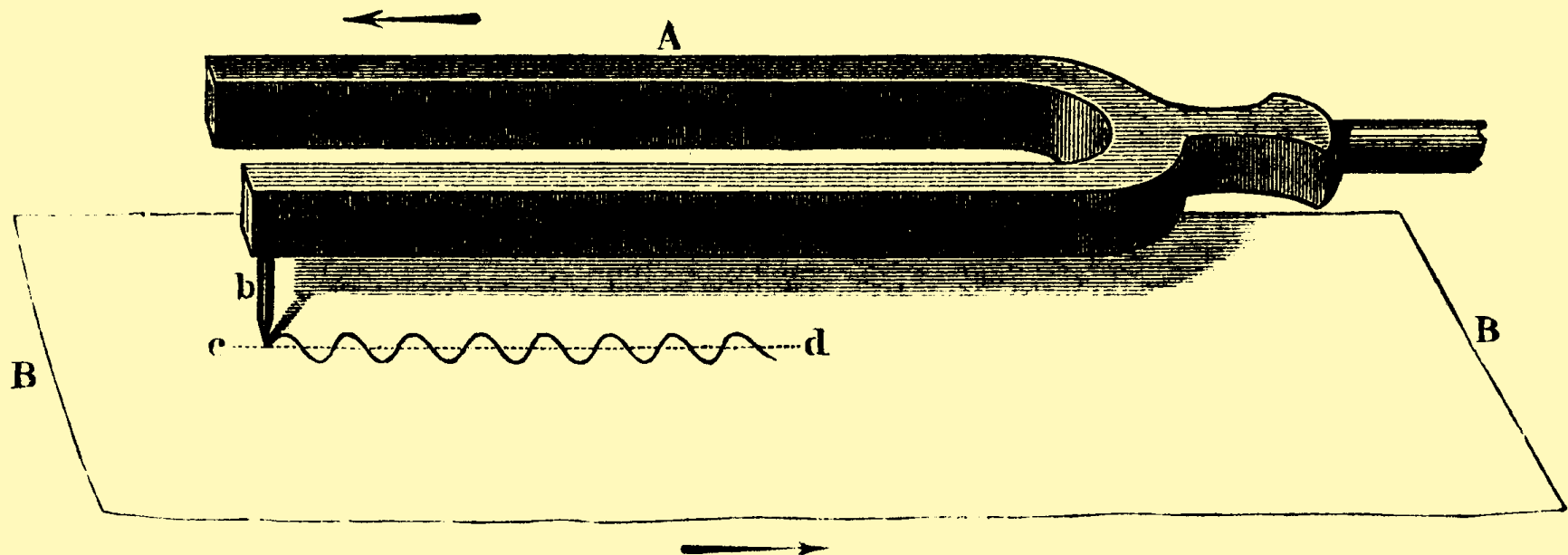
Hermann Helmholtz,
On the Sensations of Tone, 1887 (Eng. Trans. 1954)

b. 1821, d. 1894

"Physicists ... having in their minds ... forms, representing the law of motion of sounding bodies, speak briefly of the form of vibration of a sounding body, and assert that the quality of tone depends on the form of vibration."

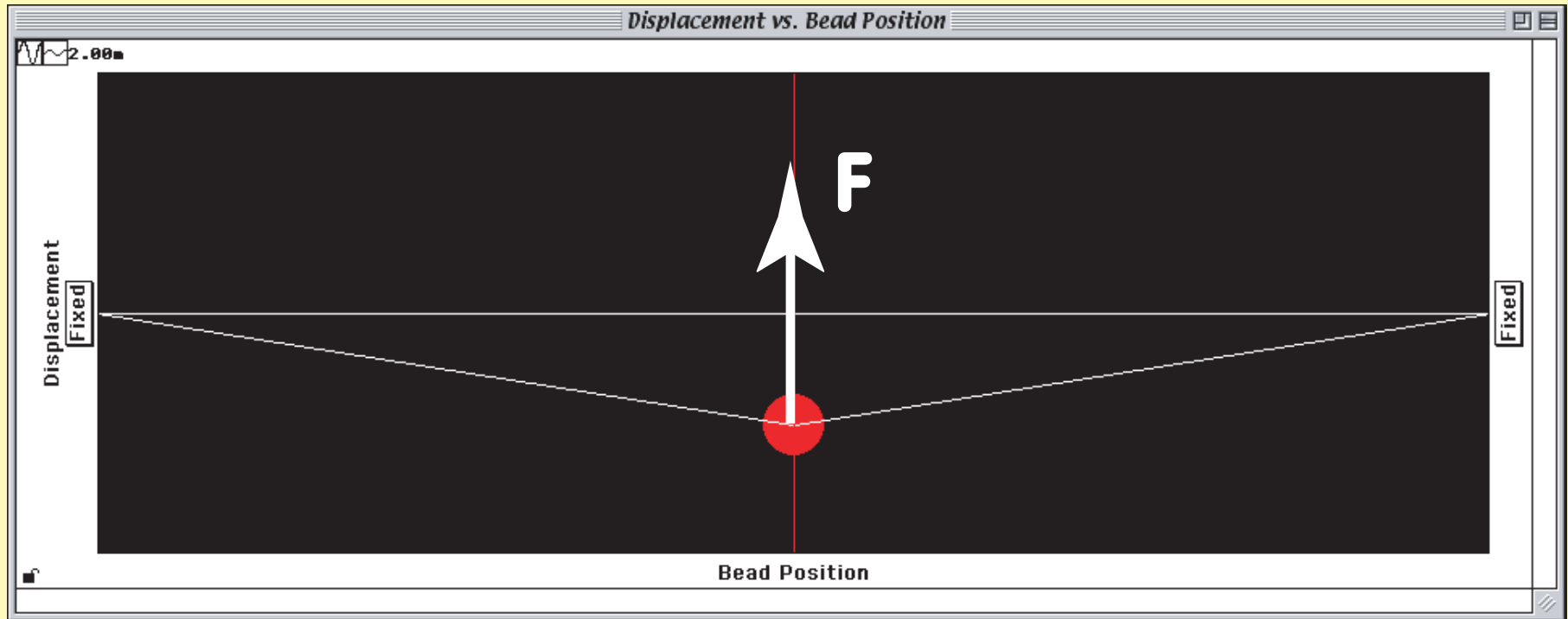
Chap. 1, p. 21

FIG. 5.



Patterns of Vibrations

Model: A bead, mass m , held under tension T , by two elastic cords, each of length L



Restoring Force:
(valid approximation for $y \ll L$,
constant tension, T)

$$F = -2Ty$$

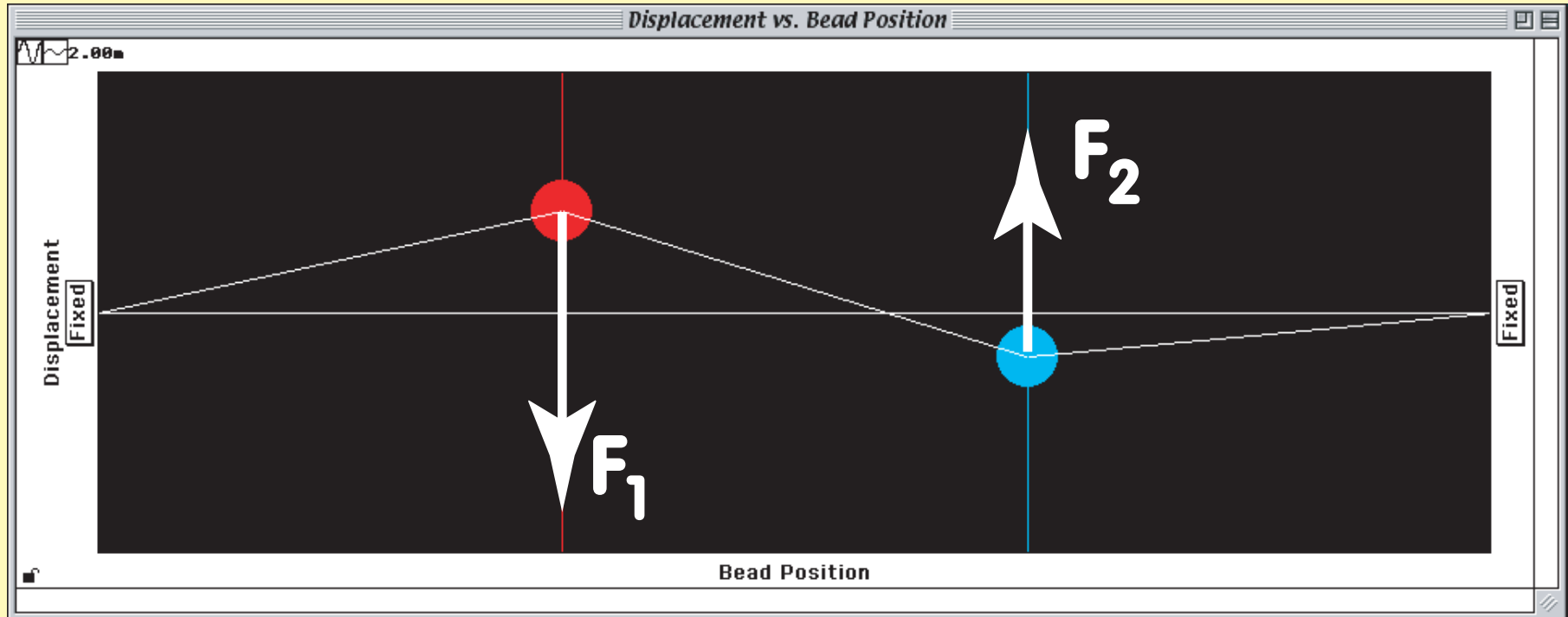
The restoring Force, F , is proportional
to the displacement, y .
This is the recipe for SHM!

Period:

$$P = 2\pi\sqrt{\frac{m}{2T}}$$

Patterns of Vibrations

What happens if we add another bead to the system and disturb in an arbitrary way?



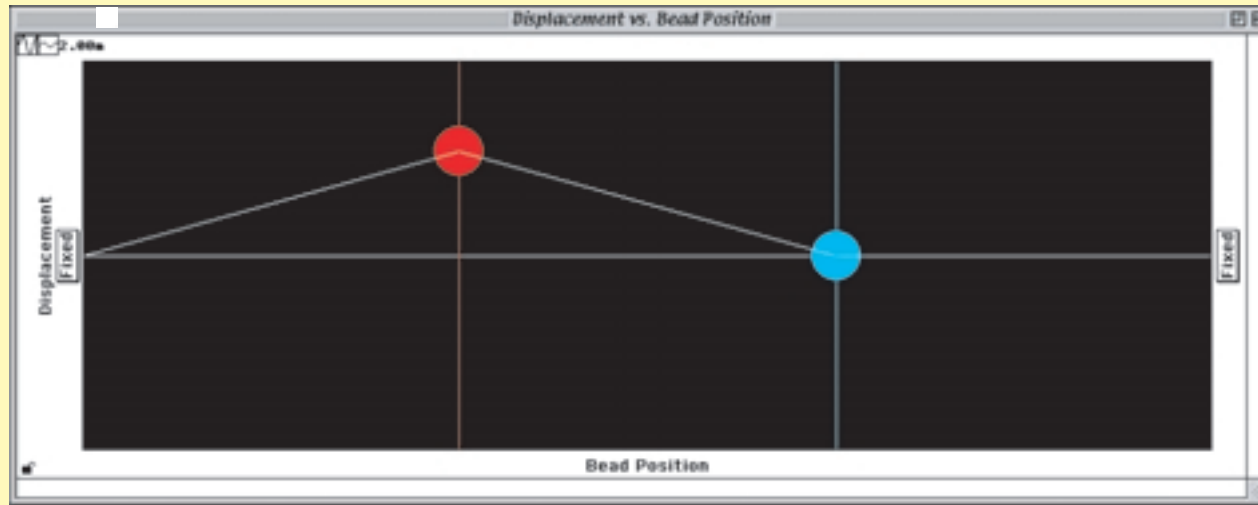
Now, both beads move, and the restoring force on each bead depends not only on its own position but also on the position of the other bead.

$$\begin{aligned} F_1 &= -Ty_1 - T(y_1 - y_2) \\ F_2 &= -Ty_2 - T(y_2 - y_1) \end{aligned}$$

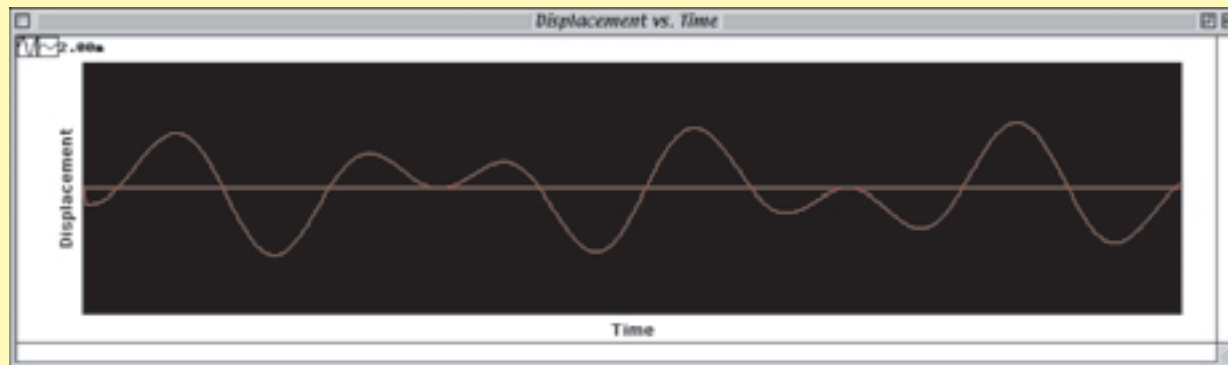
In this case, the motion of each bead is more complicated than that of one bead only

Patterns of Vibrations

The red bead is displaced, then released



The resulting motion of the red bead is not SHM! It's a more complex oscillation



Natural Modes-Natural Frequencies

Even though the motion of our 2 bead system can be complex, we can still use the idea of SHM to describe what's going on.

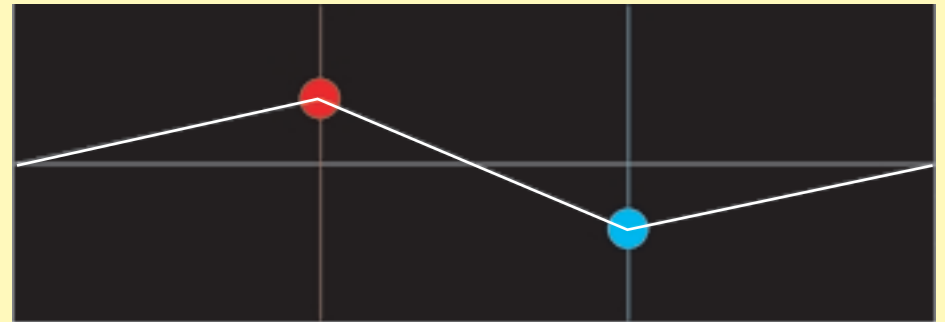
It turns out that our 2 bead system can exhibit simple harmonic motion if it is started out the right way. In fact, there are exactly two patterns of vibration in which our 2 bead system can exhibit SHM. These special patterns of vibration are termed *Natural Modes*. The frequencies of the Natural Modes are called *Natural Frequencies*

Mode 1: (Symmetric-beads move together)



Frequency: f_1

Mode 2: (Antisymmetric-beads move contrary)



Frequency: $f_2 = 1.7 f_1$
(for evenly spaced equal masses)

Typically, higher order modes have higher natural frequencies

Natural Modes-Natural Frequencies

The idea of Natural modes and Natural frequencies of vibration carries through to even more complex systems.

5 evenly spaced equal masses -----> 5 Natural Modes, with patterns and frequencies as below

Mode 1:

frequency: $f_1 = f_1$

(1 antinode, 0 nodes)



Mode 2:

frequency: $f_2 = 1.96 f_1$

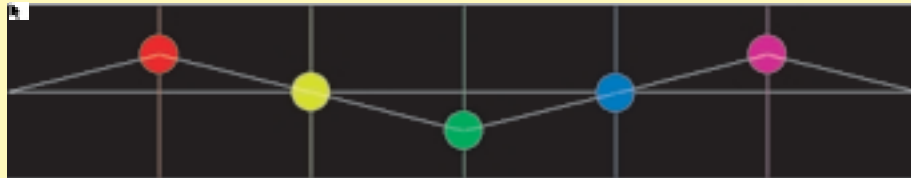
(2 antinodes, 1 node)



Mode 3:

frequency: $f_3 = 2.76 f_1$

(3 antinodes, 2 nodes)



Mode 4:

frequency: $f_4 = 3.40 f_1$

(4 antinodes, 3 nodes)



Mode 5:

frequency: $f_5 = 3.78 f_1$

(5 antinodes, 4 nodes)



Natural Modes-Natural Frequencies

General Rules

1. *Degrees of Freedom:* A mechanical system of point masses has N *degrees of freedom*, i.e. N possible independent motions. N is the product of the number of masses in the system times the number of independent directions each mass is allowed to move.
2. *Natural Modes:* A mechanical system of N degrees of freedom has exactly N *Natural Modes*, in which the every part of the system undergoes SHM. Any point of the system that remains at rest during the natural mode motion is called a *node* of that mode
3. *Characteristic Frequency:* Each Natural Mode has its own *characteristic frequency*, and usually all N frequencies are different. These frequencies can be determined mathematically in many cases. Each mode has its own characteristic decay rate, with the higher frequency modes typically decaying faster than the lower frequency modes.
4. Any vibration whatsoever in a system with N degrees of freedom can be accounted for as a superposition of the N natural modes. There are mathematical procedures that can tell us the mode recipe for any vibration of the system.

Striking points & Vibration recipes

- ▶ Each mode is excited more or less, according to how closely the pattern of the disturbing force matches the pattern of the mode itself.
- ▶ Striking an object at any given point excites each natural mode in proportion to how much that mode involves motion of that particular point.
- ▶ If the striking point is on a nodal point or line of some particular mode, then that mode is completely left out of the recipe.
- ▶ Striking an entire region at once instead of just a point produces the same recipe as you get by adding together the recipes for striking at each of the points contained in that region.
- ▶ If a striking force has finite duration T in time, then only modes whose frequencies are less than approximately $2/T$ are efficiently excited.