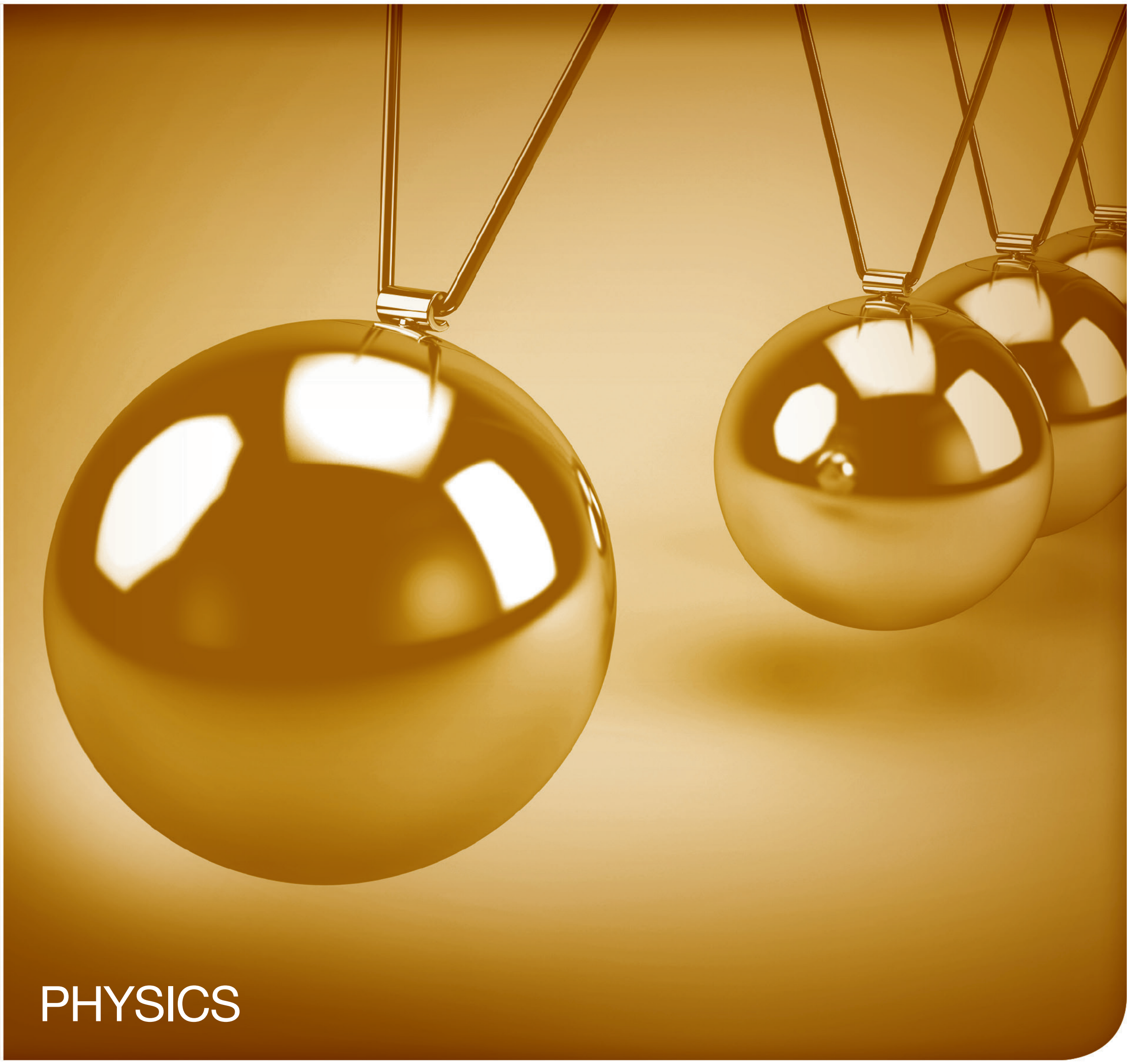

PHYSICS

Projectile Motion

Investigation Manual

CAROLINA DISTANCE LEARNING

PROJECTILE MOTION

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Overview

In this investigation, students study projectile motion by exploring two-dimensional motion and how vectors are used to describe the trajectory of an object. They will observe the motion of an object launched horizontally at various speeds and will learn how to predict the motion of the launched object by combining their prior knowledge of kinematics with new knowledge of vectors and the trajectory of projectiles.

Outcomes

- Describe what factors affect the trajectory of a projectile.
- Explain how vectors are used to describe two-dimensional and projectile motion.
- Predict the trajectory of a horizontally launched projectile using vectors and kinematic equations.

Time Requirements

Preparation .... 15 minutes
Activity 1: Launching a Projectile in a
Horizontal Direction .... 30 minutes

Key

Personal protective equipment (PPE)

goggles gloves apron

follow link to video

photograph results and submit

stopwatch required

warning corrosion flammable toxic environment health hazard

Made ADA compliant by NetCentric Technologies using the CommonLook® software

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Background

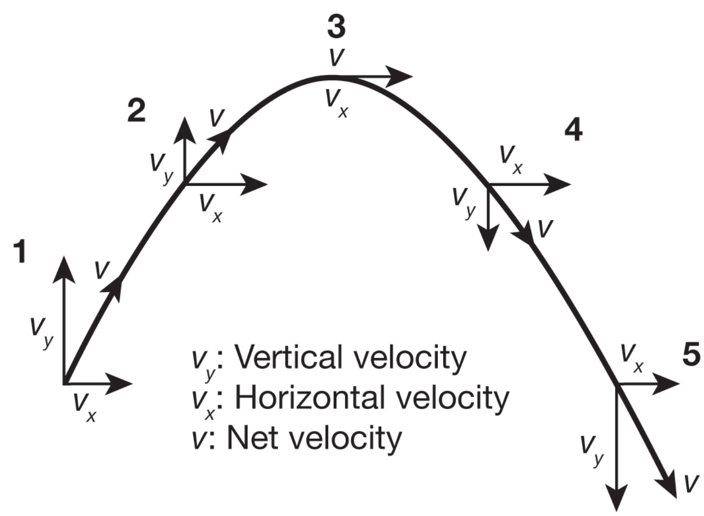
Projectiles are objects that are given an initial velocity and subsequently travel along their **trajectory** (flight path) due to their own inertia. In sports, a projectile is a basketball that has been thrown through a hoop, a pitched baseball, or a golf ball that has been hit by a golf club. At a circus, a clown launched from a cannon or a trapeze artist soaring through the air are examples of projectile motion.

Vectors describe the velocity, acceleration, and forces that act upon a projectile in terms of direction and magnitude. The principles of vector addition are used to understand and predict the trajectory of projectiles as well as other applications of two-dimensional motion, such as circular motion or the elliptical orbits of planets and comets. Therefore, vector addition is an important subject in the field of mechanics—a branch of physics that studies how physical bodies behave when subjected to forces or displacements.

To understand the motion of a projectile, you need to know the initial parameters. You can perform many calculations on a projectile's trajectory to find things such as maximum height, the time of flight, and the range (horizontal distance) the object will travel. For example, a ball thrown with less force has a lower speed and hits the ground sooner and nearer than the same ball thrown with greater force. However, the angle at which the ball is thrown also affects the trajectory of the ball. Which matters more: the initial speed or the release angle? What happens when a ball is thrown at a high speed but at a shallow angle? Will it travel farther than a ball traveling at a low speed at a greater angle? The answers to these questions can all be calculated by applying kinematic equations and some knowledge about vectors.

Projectiles tend to follow a **parabolic trajectory**. If you draw a line that follows the movement of a ball after you throw it, you would see the shape of a parabola. The shape of the parabola depends on the initial speed and the release angle, but all projectiles launched at an angle follow this parabolic curve (see Figure 1).

Figure 1.



To understand the motion of a projectile, it helps to consider the object as moving in two dimensions: the vertical *(y)* direction and the horizontal *(x)* direction. The velocity of the projectile at any given time can be broken down or resolved into a vector in the x direction and a vector in the y direction. The magnitudes of these vectors are independent of one another. Gravity only affects the vertical component of the velocity, not the horizontal component.

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PROJECTILE MOTION

Background continued

Consider Figure 1. When the projectile is launched, the velocity, $\boldsymbol{v}$, consists of two independent, perpendicular components: $\boldsymbol{v}_x$ and $\boldsymbol{v}_y$. If air resistance is negligible, the horizontal component of the velocity ($\boldsymbol{v}_x$) remains constant, whereas the vertical component of the velocity ($\boldsymbol{v}_y$) changes due to gravitational acceleration. The initial value for $\boldsymbol{v}_y$ decreases as the projectile travels to the highest point in the parabolic arc and then increases in the opposite direction as the projectile descends. If air resistance is negligible, the vertical velocity of the projectile when it returns to the elevation from which it was launched will have the same magnitude as when the projectile was launched, but the direction will have turned 180°.

Consider two projectiles launched horizontally at exactly the same time and from the same height, but one projectile has an initial velocity that is twice the other projectile. If the ground beneath the projectiles is level and air resistance is ignored, both projectiles will land on the ground at the same time. This may seem counterintuitive, because the projectile with the greater speed is traveling farther. But experimentation proves that the time of flight of both projectiles will be the same and that both projectiles will land at the same time. The projectile with the greater velocity will land farther and its parabolic trajectory will be different, but the time for the two projectiles to reach the ground will be the same.

When air resistance is taken into account, the mathematics describing the motion of projectiles can be challenging, but in many cases the air resistance is negligible and can be ignored. If air resistance is ignored, the motion of a projectile can be described by kinematic equations. The motion in the horizontal direction is constant and can be described with this simple equation:

$$\boldsymbol{v}_x = \frac{x}{t}$$

where

- $\boldsymbol{v}_x$ is the magnitude of the horizontal component of the projectile’s velocity
- x is the horizontal displacement that the object travels
- t is the time

Although the projectile’s velocity in the horizontal direction is constant, its velocity in the y direction is constantly being accelerated by gravity at a rate of $\boldsymbol{g} = 9.8\ m/s^2$.

If a projectile is fired at an angle of 0° from the horizontal (i.e., in the x direction), the time for the projectile to fall to the ground depends only on the initial height and the acceleration due to gravity. The time is independent of the horizontal velocity.

The motion of the projectile in the y direction, which is affected due to the acceleration of gravity, can be described by these kinematic equations:

$$y = \frac{1}{2}\boldsymbol{a}t^2 + \boldsymbol{v}_{y1}\Delta t$$

$$\boldsymbol{v}_{y2}{}^2 = \boldsymbol{v}_{y1}{}^2 + 2\boldsymbol{a}y$$

$$\boldsymbol{v}_{y2} = \boldsymbol{v}_{y1} + \boldsymbol{a}\Delta t$$

$$y = \frac{1}{2}(\boldsymbol{v}_{y1} + \boldsymbol{v}_{y2})\Delta t$$

where

- y is the displacement of the projectile in the y direction

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- ***a*** is the acceleration in the *y* direction, which in this context is equal to the acceleration due to gravity, $\boldsymbol{g} = 9.8\ m/s^2$
- $\boldsymbol{v}_{y2}$ is the velocity of the object in the *y* direction at time t_2
- $\boldsymbol{v}_{y1}$ is the velocity in the *y* direction at time t_1
- Δt is the time of flight between t_1 and t_2

Because the magnitudes of perpendicular vectors are independent of each other, you can calculate the time that a projectile travels by considering only the vertical component of the velocity. Once you know the time of flight for the projectile, calculate the horizontal distance that the object travels by multiplying this time by the horizontal speed of the projectile.

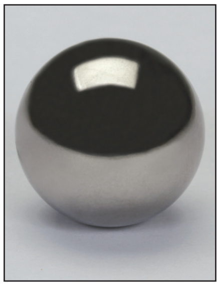
In Activity 1, you will predict and then measure the horizontal distance of a projectile launched from an elevated position with an initial velocity that has only a horizontal component. To measure the horizontal distance that the projectile travels, you will need to know the horizontal speed of the projectile and the time that the projectile is in the air.

The projectile in this activity is the metal sphere from the conceptual physics mechanics module kit. The sphere will roll down an incline using the angle bar as a track, then transition to a grooved ruler so that it travels horizontally when it leaves the table. You will apply your knowledge of kinematics to determine the velocity of the sphere as it leaves the table.

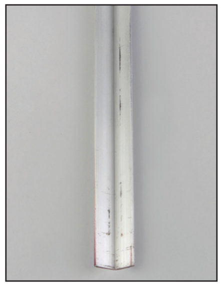
Since the sphere has no vertical velocity as it leaves the table, you will determine the time for the sphere to reach the ground by the height of the table and the acceleration due to gravity ($\boldsymbol{g} = 9.8\ m/s^2$).

Materials

Included in the conceptual physics mechanics module kit


Metal sphere


Acrylic sphere


Angle bar

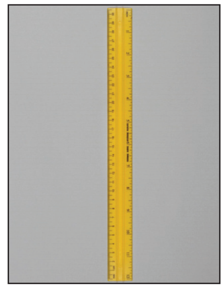

Clay

Needed but not supplied:
- Book
- Masking tape
- Calculator
- Table (or other level surface)
- Digital device capable of recording videos (optional)

Reorder Information: Replacement supplies for the Projectile Motion investigation (Conceptual Physics Mechanics Module kit, item number 580404) can be ordered from Carolina Biological Supply Company.

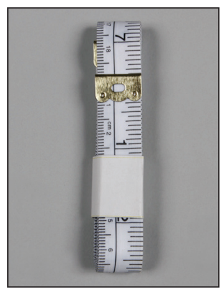
Call: 800.334.5551 to order.

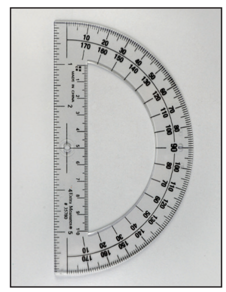
Needed from the central materials set:


Yellow grooved ruler


String


Washer


Tape measure


Protractor

Safety

Safety goggles should be worn at all times during this activity, which involves the movement and acceleration of objects. Take care during the execution of this activity to avoid injury.

Read all the instructions for this investigation before beginning. Follow the instructions closely, and observe established safety practices, including the use of appropriate personal protective equipment.

Do not eat, drink, or chew gum while performing this activity. Wash your hands with soap and water before and after performing the activity. Clean the work area with soap and water after completing the investigation. Keep pets and children away from lab materials and equipment.

Technology

Alternate Methods for Collecting Data Using Digital Devices

Much of the uncertainty in physics experiments arises from human reaction time error in measuring the times of events. Some of the time intervals are very short, which increases the effect of human error due to reaction time.

Observing the experiment from a good vantage point that removes parallax errors and recording measurements for multiple trials helps to minimize error; using a digital device as an alternate method of data collection may further minimize error. Many digital devices, such as smartphones and tablets, have cameras and software that allow the user to pause or slow down the video. If you film the activity against a scale such as a tape measure, you can use your video playback program to record position and time data. This can provide more accurate data and may eliminate the need for multiple trials.

If the time on your device's playback program is not sufficiently accurate, you may download an app such as the following free apps:

- **Hudl Technique**
  iOS and Android (https://www.hudl.com/products/technique)
- **SloPro**
  iOS (https://itunes.apple.com/us/app/slopro-1000fps-slow-motion/id507232505?mt=8)

  Android (https://slopro.en.uptodown.com/android)

Or you may upload the video to your computer. Your operating system or software suite may include video playback programs, or these programs may be available for download.

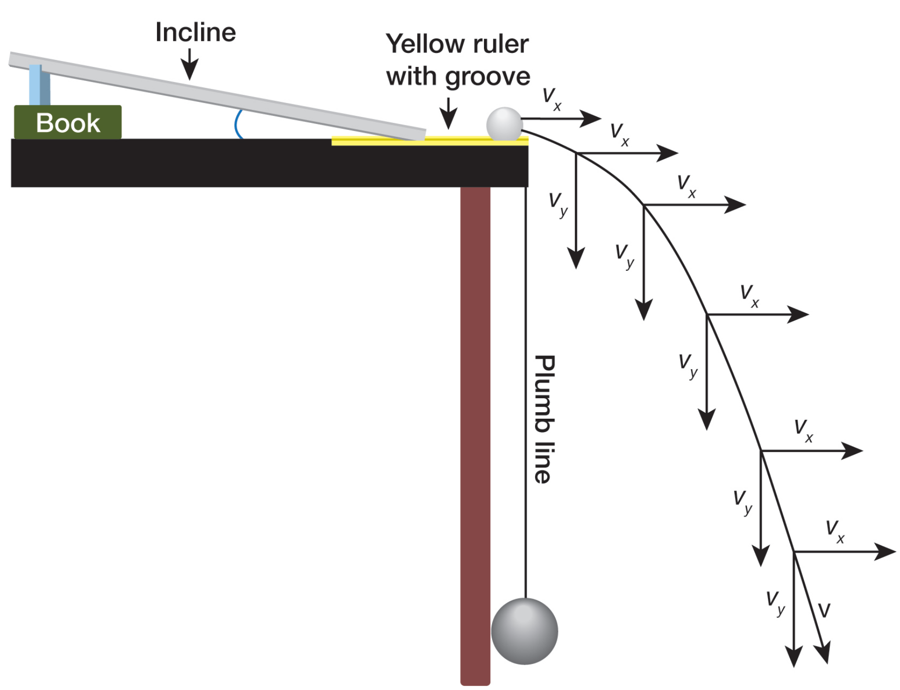
Preparation

1. Locate a smooth table at least 70 centimeters above the floor.
2. Clear the table and the floor in front of it.
3. Position the book on the table so when one end of the angle bar rests on the book the other end stops about 5 centimeters from the table's edge (see Figure 2).
4. Place some clay on the book to create a seat for the angle bar.
5. Place the yellow grooved ruler at the end of the angle bar so that the angle bar rests in the groove and the end of the ruler aligns with the table's edge.

> **Note:** For this activity, the sphere must roll down the angle bar and leave the table with a horizontal velocity. The groove in the ruler allows the sphere to transition from the incline to a horizontal direction.

6. Tape the ruler to the table to secure it, placing the tape behind the point where the angle bar rests on the ruler so that the tape does not interfere with the sphere as it rolls.
7. On the edge of the table just below the end of the ruler, tape a piece of string so the string hangs vertically from the table. The string should stop about 3 centimeters from the floor.
8. Tie the washer to the bottom end of the string. This is a plumb line and will allow you to find the point on the floor directly below the point where the sphere will leave the table.
9. Measure the angle of the angle bar vs. the table with the protractor (see Figure 2). Record the value in **Data Table 1** in column θ for trial 1.
10. Mark a point about 3 centimeters from the higher end of the angle bar. This will be the start point.

Figure 2.



ACTIVITY 1

A Launching a Projectile in a Horizontal Direction

1. Using the tape measure, measure straight down from the top of the table to the floor. Follow the plumb line to make sure the tape measure is straight.
2. Rearrange the kinematic equation for vertical displacement ($\Delta \boldsymbol{x}$) to write an equation for time.

$$\Delta \boldsymbol{x} = \frac{1}{2}\boldsymbol{a}\Delta t^2$$

$$t = \sqrt{\frac{2\Delta \boldsymbol{x}}{\boldsymbol{a}}}$$

Because the sphere is in free-fall after it leaves the table, the acceleration will be equal to gravitational acceleration:

$$\boldsymbol{a} = \boldsymbol{g} = 9.8\frac{m}{s^2}$$

The displacement x is the vertical height from the table to the floor.

$$x = h$$

Therefore, the equation for the time of flight, t, can be rewritten:

$$t = \sqrt{\frac{2h}{\boldsymbol{g}}}$$

Calculate the value for time using this equation, and write the value in **Data Table 1**. This time should be the same for each trial.

3. Calculate the horizontal velocity the sphere will have as it leaves the table by calculating the velocity of the sphere at the bottom of the incline. First, determine the acceleration of the sphere as it rolls down the incline. The acceleration of the sphere as it rolls is given by:

$$\boldsymbol{a} = 0.71\boldsymbol{g} \sin \theta$$

Substitute the angle of the incline for θ, and record the value for acceleration in **Data Table 1**.

4. Use the value for acceleration to find the horizontal speed of the sphere as it leaves the table by applying the following kinematic equation:

$$\boldsymbol{v}_x^{\,2} = \boldsymbol{v}_1^{\,2} + 2\boldsymbol{a}\Delta \boldsymbol{x}$$

- $\boldsymbol{v}_x$ is the translational velocity of the sphere as it reaches the bottom of the angle bar.
- $\boldsymbol{v}_1$ is the initial velocity of the sphere.
- $\boldsymbol{a}$ is the acceleration of the sphere.
- $\Delta \boldsymbol{x}$ is the length of the angle bar from the start point to the end of the slope.

Assume the sphere travels at this speed along the length of the horizontal ruler. Rearrange the equation, and substitute the value for $\boldsymbol{a}$ (acceleration) from Step 3 and the length of the angle bar from the start point to the end of the ramp.

$$\boldsymbol{v}_x = \sqrt{2\boldsymbol{a}\Delta \boldsymbol{x}}$$

Record the value for $\boldsymbol{v}_x$ in **Data Table 1**.

5. Multiply the value for the horizontal velocity ($\boldsymbol{v}_x$) by the time (found in Step 2). Record the value (in meters) in **Data Table 1**. This is the distance that the sphere will travel before it strikes the floor.
6. Using the tape measure, find the point on the floor that is at the same distance from the table as the value calculated in Step 5. Measure from directly beneath the plumb line and in the same direction that the angle bar is pointing.

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ACTIVITY 1 continued

7. Place the sphere at the start point on the high end of the angle bar.
8. Release the metal sphere, and allow it to roll down the angle bar, across the grooved ruler, and off the table. The sphere should land on or close to the point you marked on the floor.
9. Measure the distance to the point where the sphere struck the floor. Record the actual distance in meters in **Data Table 1**.
10. Find the percent error between the distance you calculated and the distance actually traveled by the sphere.

$$\text{percent error} = \frac{|\text{calculated} - \text{experimental}|}{\text{calculated}} \times 100\%$$

Record your results in **Data Table 1.**

11. Repeat the experiment with the acrylic sphere.
12. Repeat the experiment using both the metal and acrylic spheres, increasing the angle by 5° and then by 10°.

Disposal and Cleanup

Return the equipment to the module kit and materials set, and clean the work area.

Observations

Data Table 1.

Trial	Sphere	θ	$\boldsymbol{a} = 0.71\ (9.8)\sin\theta$	$\boldsymbol{v}_x = \sqrt{2\boldsymbol{a}\Delta \boldsymbol{x}}$	$t = \sqrt{\frac{2h}{\boldsymbol{g}}}$	Calculated Distance $x = \boldsymbol{v}_x t$ (meters)	Actual Distance (meters)	Percent Error
1	Metal							
2	Acrylic							
3	Metal	+ 5°						
4	Acrylic	+ 5°						
5	Metal	+ 10°						
6	Acrylic	+ 10°						

NOTES

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