

From
Readings in Philosophical Analysis

ed. by Herbert Feigl and
Wilfrid Sellars.

Appleton-Century-Crofts, 1949.

On Sense and Nominatum *

GOTTLOB FREGE

The idea of Sameness¹ challenges reflection. It raises questions which are not quite easily answered. Is Sameness a relation? A relation between objects? Or between names or signs of objects? I assumed the latter alternative in my *Begriffsschrift*. The reasons that speak in its favor are the following: "a = a" and "a = b" are sentences of obviously different cognitive significance: "a = a" is valid *a priori* and according to Kant is to be called analytic, whereas sentences of the form "a = b" often contain very valuable extensions of our knowledge and cannot always be justified in an *a priori* manner. The discovery that it is not a different and novel sun which rises every morning, but that it is the very same, certainly was one of the most consequential ones in astronomy. Even nowadays the re-cognition (identification) of a planetoid or a comet is not always a matter of self-evidence. If we wished to view identity as a relation between the objects designated by the names 'a' and 'b' then "a = b" and "a = a" would not seem different if "a = b" is true. This would express a relation of a thing to itself, namely, a relation such that it holds between every thing and itself but never between one thing and another. What one wishes to express with "a = b" seems to be that the signs or names 'a' and 'b' name the same thing; and in that case we would be dealing with those signs: a relation between them would be asserted. But this relation could hold only inasmuch as they name or designate something. The relation, as it were, is mediated through the connection of each sign with the same nominatum. This connection, however, is arbitrary. You cannot forbid the use of an arbitrarily produced process or object as a sign for something else. Hence, a sentence like "a = b" would no longer refer to a matter of fact but rather to our manner of designation; no genuine knowledge would be expressed by it. But this is just what we do want to express in many cases. If the sign 'a' differs from the sign 'b' only as an object (here by its shape) but not by its rôle as a sign, that is to say, not in the manner in which it designates anything, then the cognitive significance of "a = a" would be essentially the same as that of

* Translated by H.F. from the article "Ueber Sinn und Bedeutung", *Zeitschr. f. Philos. und Philos. Kritik*; 100, 1892. The terminology adopted is largely that used by R. Carnap in *Meaning and Necessity*, Univ. of Chicago Press, 1947.

¹I use this word in the sense of identity and understand "a = b" in the sense of "a is the same as b" or "a and b coincide".

" $a = b$ ", if " $a = b$ " is true. A difference could arise only if the difference of the signs corresponds to a difference in the way in which the designated objects are given. Let a , b , c be straight lines which connect the corners of a triangle with the midpoints of the opposite sides. The point of intersection of a and b is then the same as that of b and c . Thus we have different designations of the same point and these names ('intersection of a and b ', 'intersection of b and c ') indicate also the manner in which these points are presented. Therefore the sentence expresses a genuine cognition.

Now it is plausible to connect with a sign (name, word combination, expression) not only the designated object, which may be called the nominatum of the sign, but also the sense (connotation, meaning) of the sign in which is contained the manner and context of presentation. Accordingly, in our examples the *nominata* of the expressions 'the point of intersection of a and b ' and 'the point of intersection of b and c ' would be the same;—not their senses. The *nominata* of 'evening star' and 'morning star' are the same but not their senses.

From what has been said it is clear that I here understand by 'sign' or 'name' any expression which functions as a proper name, whose nominatum accordingly is a definite object (in the widest sense of this word). But no concept or relation is under consideration here. These matters are to be dealt with in another essay. The designation of a single object may consist of several words or various signs. For brevity's sake, any such designation will be considered as a proper name.

The sense of a proper name is grasped by everyone who knows the language or the totality of designations of which the proper name is a part;² this, however, illuminates the nominatum, if there is any, in a very one-sided fashion. A complete knowledge of the nominatum would require that we could tell immediately in the case of any given sense whether it belongs to the nominatum. This we shall never be able to do.

The regular connection between a sign, its sense and its nominatum is such that there corresponds a definite sense to the sign and to this sense there corresponds again a definite nominatum; whereas not one sign only belongs to one nominatum (object). In different languages, and even in one language, the same sense is represented by different expressions. It is true, there are exceptions to this rule. Certainly there should be a definite sense to each expression in a complete configuration of signs, but the natural languages in many ways fall short of this requirement. We must be satisfied if the same word, at least in the same context, has the same

² In the case of genuinely proper names like 'Aristotle' opinions as regards their sense may diverge. As such may, e. g., be suggested: Plato's disciple and the teacher of Alexander the Great. Whoever accepts this sense will interpret the meaning of the statement "Aristotle was born in Stagira" differently from one who interpreted the sense of 'Aristotle' as the Stagirite teacher of Alexander the Great. As long as the nominatum remains the same, these fluctuations in sense are tolerable. But they should be avoided in the system of a demonstrative science and should not appear in a perfect language.

sense. It can perhaps be granted that an expression has a sense if it is formed in a grammatically correct manner and stands for a proper name. But as to whether there is a denotation corresponding to the connotation is hereby not decided. The words 'the heavenly body which has the greatest distance from the earth' have a sense; but it is very doubtful as to whether they have a nominatum. The expression 'the series with the least convergence' has a sense; but it can be proved that it has no nominatum, since for any given convergent series, one can find another one that is less convergent. Therefore the grasping of a sense does not with certainty warrant a corresponding nominatum.

When words are used in the customary manner then what is talked about are their *nominata*. But it may happen that one wishes to speak about the words themselves or about their senses. The first case occurs when one quotes someone else's words in direct (ordinary) discourse. In this case one's own words immediately name (denote) the words of the other person and only the latter words have the usual *nominata*. We thus have signs of signs. In writing we make use of quotes enclosing the word-*icons*. A word-*icon* in quotes must therefore not be taken in the customary manner.

If we wish to speak of the sense of an expression 'A' we can do this simply through the locution 'the sense of the expression 'A''. In indirect (oblique) discourse we speak of the sense, e. g., of the words of someone else. From this it becomes clear that also in indirect discourse words do not have their customary *nominata*; they here name what customarily would be their sense. In order to formulate this succinctly we shall say: words in indirect discourse are used *indirectly*, or have *indirect* *nominata*. Thus we distinguish the *customary* from the *indirect* nominatum of a word; and similarly, its *customary* sense from its *indirect* sense. The indirect nominatum of a word is therefore its customary sense. Such exceptions must be kept in mind if one wishes correctly to comprehend the manner of connection between signs, senses and *nominata* in any given case.

Both the nominatum and the sense of a sign must be distinguished from the associated image. If the nominatum of a sign is an object of sense perception, my image of the latter is an inner picture³ arisen from memories of sense impressions and activities of mine, internal or external. Frequently this image is suffused with feelings; the definiteness of its various parts may vary and fluctuate. Even with the same person the same sense is not always accompanied by the same image. The image is subjective; the image of one person is not that of another. Hence, the

³ With the images we can align also the percepts in which the sense impressions and activities themselves take the place of those traces left in the mind. For our purposes the difference is unimportant, especially since besides sensations and activities recollections of such help in completing the intuitive presentation. 'Percept' may also be understood as the object, inasmuch as it is spatial or capable of sensory apprehension.

various differences between the images connected with one and the same sense. A painter, a rider, a zoölogist probably connect very different images with the name 'Bucephalus'. The image thereby differs essentially from the connotation of a sign, which latter may well be common property of many and is therefore not a part or mode of the single person's mind; for it cannot well be denied that mankind possesses a common treasure of thoughts which is transmitted from generation to generation.⁴

While, accordingly, there is no objection to speak without qualification of the sense in regard to images, we must, to be precise, add *whose* images they are and at what time they occur. One might say: just as words are connected with different images in two different persons, the same holds of the senses also. Yet this difference would consist merely in the manner of association. It does not prevent both from apprehending the same sense, but they cannot have the same image. *Si duo idem faciunt, non est idem*. When two persons imagine the same thing, each still has his own image. It is true, occasionally we can detect differences in the images or even in the sensations of different persons. But an accurate comparison is impossible because these images cannot be had together in one consciousness.

The nominatum of a proper name is the object itself which is designated thereby; the image which we may have along with it is quite subjective; the sense lies in between, not subjective as is the image, but not the object either. The following simile may help in elucidating these relationships. Someone observes the moon through a telescope. The moon is comparable with the nominatum; it is the object of the observation which is mediated through the real image projected by the object lens into the interior of the telescope, and through the retinal image of the observer. The first may be compared with the sense, the second with the presentation (or image in the psychological sense). The real image inside the telescope, however, is relative; it depends upon the standpoint; yet, it is objective in that it can serve several observers. Arrangements could be made such that several observers could utilize it. But every one of them would have only his own retinal image. Because of the different structures of the eyes not even geometrical congruence could be attained; a real coincidence would in any case be impossible. One could elaborate the simile by assuming that the retinal image of *A* could be made visible to *B*; or *A* could see his own retinal image in a mirror. In this manner one could possibly show how a presentation itself can be made into an object; but even so, it would never be to the (outside) observer what it is to the one who possesses the image. However, these lines of thought lead too far afield.

We can now recognize three levels of differences of words, expressions

⁴ It is therefore inexpedient to designate fundamentally different things by the one word 'image' (or 'idea').

and complete sentences. The difference may concern at most the imagery, or else the sense but not the nominatum, or finally also the nominatum. In regard to the first level, we must note that, owing to the uncertain correlation of images with words, a difference may exist for one person that another does not discover. The difference of a translation from the original should properly not go beyond the first level. Among the differences possible in this connection we mention the shadings and colorings which poetry seeks to impart to the senses. These shadings and colorings are not objective. Every listener or reader has to add them in accordance with the hints of the poet or speaker. Surely, art would be impossible without some kinship among human imageries; but just how far the intentions of the poet are realized can never be exactly ascertained.

We shall henceforth no longer refer to the images and picturizations; they were discussed only lest the image evoked by a word be confused with its sense or its nominatum.

In order to facilitate brief and precise expression we may lay down the following formulations:

A proper name (word, sign, sign-compound, expression) expresses its sense, and designates or signifies its nominatum. We let a *sign* express its sense and *designate* its nominatum.

Perhaps the following objection, coming from idealistic or skeptical quarters, has been kept in abeyance for some time: "You have been speaking without hesitation of the moon as an object; but how do you know that the name 'the moon' has in fact a nominatum? How do you know that anything at all has a nominatum?" I reply that it is not our intention to speak of the image of the moon, nor would we be satisfied with the sense when we say 'the moon'; instead, we presuppose a nominatum here. We should miss the meaning altogether if we assumed we had reference to images in the sentence "the moon is smaller than the earth". Were this intended we would use some such locution as 'my image of the moon'. Of course, we may be in error as regards that assumption, and such errors have occurred on occasion. However, the question whether we could possibly always be mistaken in this respect may here remain unanswered; it will suffice for the moment to refer to our intention in speaking and thinking in order to justify our reference to the nominatum of a sign; even if we have to make the proviso: if there is such a nominatum.

Thus far we have considered sense and nominatum only of such expressions, words and signs which we called proper names. We are now going to inquire into the sense and the nominatum of a whole declarative sentence. Such a sentence contains a proposition.⁵ Is this thought to be regarded as the sense or the nominatum of the sentence? Let us for the

⁵ By 'proposition' I do not refer to the subjective activity of thinking but rather to its objective content which is capable of being the common property of many.

moment assume that the sentence has a nominatum! If we then substitute a word in it by another word with the same nominatum but with a different sense, then this substitution cannot affect the nominatum of the sentence. But we realize that in such cases the proposition is changed; e. g., the proposition of the sentence "the morning star is a body illuminated by the sun" is different from that of "the evening star is a body illuminated by the sun". Someone who did not know that the evening star is the same as the morning star could consider the one proposition true and the other false. The proposition can therefore not be the nominatum of the sentence; it will instead have to be regarded as its sense. But what about the nominatum? Can we even ask this question? A sentence as a whole has perhaps only sense and no nominatum? It may in any case be expected that there are such sentences, just as there are constituents of sentences which do have sense but no nominatum. Certainly, sentences containing proper names without nominata must be of this type. The sentence "Odysseus deeply asleep was disembarked at Ithaca" obviously has a sense. But since it is doubtful as to whether the name 'Odysseus' occurring in this sentence has a nominatum, so it is also doubtful that the whole sentence has one. However, it is certain that whoever seriously regards the sentence either as true or as false also attributes to the name 'Odysseus' a nominatum, not only a sense; for it is obviously the nominatum of this name to which the predicate is either ascribed or denied. He who does not acknowledge the nominatum cannot ascribe or deny a predicate to it. It might be urged that the consideration of the nominatum of the name is going farther than is necessary; one could be satisfied with the sense, if one stayed with the proposition. If all that mattered were only the sense of the sentence (i. e., the proposition) then it would be unnecessary to be concerned with the nominata of the sentence-components, for only the sense of the components can be relevant for the sense of the sentence. The proposition remains the same, no matter whether or not the name 'Odysseus' has a nominatum. The fact that we are at all concerned about the nominatum of a sentence-component indicates that we generally acknowledge or postulate a nominatum for the sentence itself. The proposition loses in interest as soon as we recognize that one of its parts is lacking a nominatum. We may therefore be justified to ask for a nominatum of a sentence, in addition to its sense. But why do we wish that every proper name have not only a sense but also a nominatum? Why is the proposition alone not sufficient? We answer: because what matters to us is the truth-value. This, however, is not always the case. In listening to an epic, for example, we are fascinated by the euphony of the language and also by the sense of the sentences and by the images and emotions evoked. In turning to the question of truth we disregard the artistic appreciation and pursue scientific considerations. Whether the name 'Odysseus' has a nominatum is therefore immaterial to us as long as we accept the poem

as a work of art.⁶ Thus, it is the striving for truth which urges us to penetrate beyond the sense to the nominatum.

We have realized that we are to look for the nominatum of a sentence whenever the nominata of the sentence-components are the thing that matters; and that is the case whenever and only when we ask for the truth value.

Thus we find ourselves persuaded to accept the *truth-value* of a sentence as its nominatum. By the truth-value of a sentence I mean the circumstance of its being true or false. There are no other truth-values. For brevity's sake I shall call the one the True and the other the False. Every declarative sentence, in which what matters are the nominata of the words, is therefore to be considered as a proper name; and its nominatum, if there is any, is either the True or the False. These two objects are recognized, even if only tacitly, by everyone who at all makes judgments, holds anything as true, thus even by the skeptic. To designate truth-values as objects may thus far appear as a capricious idea or as a mere play on words, from which no important conclusion should be drawn. What I call an object can be discussed only in connection with the nature of concepts and relations. That I will reserve for another essay. But this might be clear even here: in every judgment⁷—no matter how obvious—a step is made from the level of propositions to the level of the nominata (the objective facts).

It may be tempting to regard the relation of a proposition to the True not as that of sense to nominatum but as that of the subject to the predicate. One could virtually say: "the proposition that 5 is a prime number is true". But on closer examination one notices that this does not say any more than is said in the simple sentence "5 is a prime number". This makes clear that the relation of a proposition to the True must not be compared with the relation of subject and predicate. Subject and predicate (interpreted logically) are, after all, components of a proposition; they are on the same level as regards cognition. By joining subject and predicate we always arrive only at a proposition; in this way we never move from a sense to a nominatum or from a proposition to its truth-value. We remain on the same level and never proceed from it to the next one. Just as the sun cannot be part of a proposition, so the truth-value, because it is not the sense, but an object, cannot be either.

If our conjecture (that the nominatum of a sentence is its truth value) is correct, then the truth-value must remain unchanged if a sentence-component is replaced by an expression with the same nominatum but

⁶ It would be desirable to have an expression for signs which have sense only. If we call them 'icons' then the words of an actor on the stage would be icons; even the actor himself would be an icon.

⁷ A judgment is not merely the apprehension of a thought or proposition but the acknowledgment of its truth.

with a different sense. Indeed, Leibnitz declares: "*Eadem sunt, quae sibi mutuo substitui possunt, salva veritate*". What else, except the truth-value, could be found, which quite generally belongs to every sentence and regarding which the nominata of the components are relevant and which would remain invariant for substitutions of the type indicated?

Now if the truth-value of a sentence is its nominatum, then all true sentences have the same nominatum, and likewise all false ones. This implies that all detail has been blurred in the nominatum of a sentence. What interests us can therefore never be merely the nominatum; but the proposition alone does not give knowledge; only the proposition together with its nominatum, i. e., its truth-value, does. Judging may be viewed as a movement from a proposition to its nominatum, i. e., its truth-value. Of course this is not intended as a definition. Judging is indeed something peculiar and unique. One might say that judging consists in the discerning of parts within the truth-value. This discernment occurs through recourse to the proposition. Every sense that belongs to a truth-value would correspond in its own manner to the analysis. I have, however, used the word 'part' in a particular manner here: I have transferred the relation of whole and part from the sentence to its nominatum. This I did by viewing the nominatum of a word as part of the nominatum of a sentence, when the word itself is part of the sentence. True enough, this way of putting things is objectionable since as regards the nominatum the whole and one part of it does not determine the other part; and also because the word 'part' in reference to bodies has a difference customary usage. A special expression should be coined for what has been suggested above.

We shall now further examine the conjecture that the truth-value of a sentence is its nominatum. We have found that the truth-value of a sentence remains unaltered if an expression within the sentence is replaced by a synonymous one. But we have as yet not considered the case in which the expression-to-be-replaced is itself a sentence. If our view is correct, then the truth-value of a sentence, which contains another sentence as a part, must remain unaltered when we substitute for the part another of the same truth-value. Exceptions are to be expected if the whole or the part are either in direct or indirect discourse; for as we have seen, in that case the nominata of the words are not the usual ones. A sentence in direct discourse nominates again a sentence but in indirect discourse it nominates a proposition.

Our attention is thus directed to subordinate sentences (i. e., dependent clauses). These present themselves of course as parts of a sentence-structure which from a logical point of view appears also as a sentence, and indeed as if it were a main clause. But here we face the question whether in the case of dependent clauses it also holds that their nominata are truth-values. We know already that this is not the case with sentences in indirect discourse. The grammarians view clauses as representatives of sentence-parts

and divide them accordingly into subjective, relative, and adverbial clauses. This might suggest that the nominatum of a clause is not a truth-value but rather that it is of similar nature as that of a noun or of an adjective or of an adverb; in short, of a sentence-part whose sense is not a proposition but only part thereof. Only a thorough investigation can provide clarity in this matter. We shall herein not follow strictly along grammatical lines, but rather group together what is logically of comparable type. Let us first seek out such instances in which, as we just surmised, the sense of a clause is not a self-sufficient proposition.

Among the abstract clauses beginning with 'that' there is also the indirect discourse, of which we have seen that in it the words have their indirect (oblique) nominata which coincide with what are ordinarily their senses. In this case then the clause has as its nominatum a proposition, not a truth-value; its sense is not a proposition but it is the sense of the words 'the proposition that . . .', which is only a part of the proposition corresponding to the total sentence-structure. This occurs in connection with 'to say', 'to hear', 'to opine', 'to be convinced', 'to infer' and similar words.⁸ The situation is different, and rather complicated in connection with such words as 'to recognize', 'to know', 'to believe', a matter to be considered later.

One can see that in these cases the nominatum of the clause indeed consists in the proposition, because whether that proposition is true or false is immaterial for the truth of the whole sentence. Compare, e. g., the following two sentences: "Copernicus believed that the planetary orbits are circles" and "Copernicus believed that the appearance of the sun's motion is produced by the real motion of the earth". Here the one clause can be substituted for the other without affecting the truth. The sense of the principal sentence together with the clause is the single proposition; and the truth of the whole implies neither the truth nor the falsity of the clause. In cases of this type it is not permissible to replace in the clause one expression by another of the same nominatum. Such replacement may be made only by expressions of the same indirect nominatum, i. e., of the same customary sense. If one were to infer: the nominatum of a sentence is not its truth-value ("because then a sentence could always be replaced by another with the same truth-value"), he would prove too much; one could just as well maintain that the nominatum of the word 'morning star' is not Venus, for one cannot always substitute 'Venus' for 'morning star'. The only correct conclusion is that the nominatum of a sentence is *not always* its truth-value, and that 'morning star' does not always nominate the planet Venus; for this is indeed not the case when the word is used with its indirect nominatum. Such an exceptional case is before us in the clauses just considered, whose nominatum is a proposition.

⁸ In "*A* lied, that he had seen *B*" the clause denotes a proposition of which it is said, firstly, that *A* asserted it as true, and, secondly, that *A* was convinced of its falsity.

When we say "it seems that . . ." then we mean to say "it seems to me that . . ." or "I opine that . . .". This is the same case over again. Similarly with expressions such as: 'to be glad', 'to regret', 'to approve', 'to disapprove', 'to hope', 'to fear'. When Wellington, toward the end of the battle of Belle-Alliance was glad that the Prussians were coming, the ground of his rejoicing was a conviction. Had he actually been deceived, he would not have been less glad, as long as his belief persisted; and before he arrived at the conviction that the Prussians were coming he could not have been glad about it, even if in fact they were already approaching.

Just as a conviction or a belief may be the ground of a sentiment, so it can also be the ground of another conviction such as in inference. In the sentence "Columbus inferred from the roundness of the earth that he could, traveling westward, reach India" we have, as nominata of its parts two propositions: that the earth is round, and that Columbus traveling westward could reach India. What matters here is only that Columbus was convinced of the one as well as of the other and that the one conviction furnishes the ground for the other. It is irrelevant for the truth of our sentence whether the earth is really round and whether Columbus could have reached India in the manner he fancied. But it is not irrelevant whether for 'the earth' we substitute 'the planet accompanied by one satellite whose diameter is larger than one-fourth of its own diameter'. Here also we deal with the indirect nominata of the words.

Adverbial clauses of purpose with 'so that', likewise belong here; obviously the purpose is a proposition; therefore: indirect nominata of the words, expressed in subjunctive form.

The clause with 'that' after 'to command', 'to request', 'to forbid' would appear in imperative form in direct discourse. Imperatives have no nominata; they have only sense. It is true, commands or requests are not propositions, but they are of the same type as propositions. Therefore the words in the dependent clauses after 'to command', 'to request', etc. have indirect nominata. The nominatum of such a sentence is thus not a truth-value but a command, a request, and the like.

We meet a similar situation in the case of dependent questions in phrases like 'to doubt if', 'not to know what'. It is easy to see that the words, here too, have to be interpreted in terms of their indirect nominata. The dependent interrogatory clauses containing 'who', 'what', 'where', 'when', 'how', 'whereby', etc. often apparently approximate closely adverbial clauses in which the words have their ordinary nominata. These cases are linguistically distinguished through the mode of the verb. In the subjunctive we have a dependent question and the indirect nominata of the words, so that a proper name cannot generally be replaced by another of the same object.

In the instances thus far considered the words in the clause had indirect nominata; this made it intelligible that the nominatum of the clause itself

is indirect, i. e., not a truth-value, but a proposition, a command, a request, a question. The clause could be taken as a noun; one might even say, as a proper name of that proposition, command, etc., in whose rôle it functions in the context of the sentence-structure.

We are now going to consider clauses of another type, in which the words do have their customary nominata although there does not appear a proposition as the sense or a truth-value as the nominatum. How this is possible will best be elucidated by examples.

"He who discovered the elliptical shape of the planetary orbits, died in misery".

If, in this example, the sense of the clause were a proposition, it would have to be expressible also in a principal sentence. But this cannot be done because the grammatical subject 'he who' has no independent sense. It merely mediates the relations to the second part of the sentence: 'died in misery'. Therefore the sense of the clause is not a complete proposition and its nominatum is not a truth-value, but Kepler. It might be objected that the sense of the whole does include a proposition as its part; namely, that there was someone who first recognized the elliptical shape of the planetary orbits; for if we accept the whole as true we cannot deny this part. Indubitably so; but only because otherwise the clause "he who discovered the elliptical shape, etc." would have no nominatum. Whenever something is asserted then the presupposition taken for granted is that the employed proper names, simple or compound, have nominata. Thus, if we assert "Kepler died in misery" it is presupposed that the name 'Kepler' designates something. However, the proposition that the name 'Kepler' designates something is, the foregoing notwithstanding, not contained in the sense of the sentence "Kepler died in misery". If that were the case the denial would not read "Kepler did not die in misery" but "Kepler did not die in misery, or the name 'Kepler' is without nominatum". That the name 'Kepler' designates something is rather the presupposition of the assertion "Kepler died in misery" as well as of its denial. Now, it is a defect of languages that expressions are possible within them, which, in their grammatical form, seemingly determined to designate an object, nevertheless do not fulfill this condition in special cases; because this depends on the truth of the sentence. Thus it depends upon the truth of the sentence "there was someone who discovered the ellipticity of the orbits" whether the clause 'he who discovered the ellipticity of the orbits' really designates an object, or else merely evokes the appearance thereof, while indeed being without nominatum. Thus it may seem as if our clause, as part of its sense, contained the proposition that there existed someone who discovered the ellipticity of the orbits. If this were so, then the denial would have to read "he who first recognized the ellipticity of the orbits did not die in misery, or there was no one who discovered the ellipticity of the

orbits." This, it is obvious, hinges upon an imperfection of language of which, by the way, even the symbolic language of analysis is not entirely free; there, also, sign compounds may occur which appear as if they designated something, but which at least hitherto are without nominatum, e. g., divergent infinite series. This can be avoided, e. g., through the special convention that the nominatum of divergent infinite series be the number 0. It is to be demanded that in a logically perfect language (logical symbolism) every expression constructed as a proper name in a grammatically correct manner out of already introduced symbols, in fact designate an object; and that no symbol be introduced as a proper name without assurance that it have a nominatum. It is customary in logic texts to warn against the ambiguity of expressions as a source of fallacies. I deem it at least as appropriate to issue a warning against apparent proper names that have no nominata. The history of mathematics has many a tale to tell of errors which originated from this source. The demagogic misuse is close (perhaps closer) at hand as in the case of ambiguous expressions. 'The will of the people' may serve as an example in this regard; for it is easily established that there is no generally accepted nominatum of that expression. Thus it is obviously not without importance to obstruct once for all the source of these errors, at least as regards their occurrence in science. Then such objections as the one discussed above will become impossible, for then it will be seen that whether a proper name has a nominatum can never depend upon the truth of a proposition.

Our considerations may be extended from these subjective clauses to the logically related relative and adverbial clauses.

Relative clauses, too, are employed in the formation of compound proper names—even if, in contradistinction to subjective clauses, they are not sufficient by themselves for this purpose. These relative clauses may be regarded as equivalent to appositions. Instead of 'the square root of 4 which is smaller than 0' we can also say 'the negative square root of 4'. We have here a case in which out of a conceptual expression a compound proper name is formed, with the help of the definite article in the singular. This is at any rate permissible when one and only one object is comprised by the concept.⁹ Conceptual expression can be formed in such a fashion that their characteristics are indicated through relative clauses as in our example through the clause 'which is smaller than 0'. Obviously, such relative clauses, just as the subjective clauses above, do not refer to a proposition as their sense nor to a truth-value as their nominatum. Their sense is only a part of a proposition, which in many cases, can be expressed by a simple apposition. As in the subjective clauses an independent sub-

⁹ According to our previous remarks such an expression should always be assured of a nominatum, e. g., through the special convention that the nominatum be the number 0 if there is no object or more than one object denoted by the expression.

ject is missing and it is therefore impossible to represent the sense of the clause in an independent principal sentence.

Places, dates and time-intervals are objects from a logical point of view; the linguistic symbol of a definite place, moment or span of time must therefore be viewed as a proper name. Adverbial clauses of space or time can then be used in the formation of such proper names in a fashion analogous to the one we have just remarked in the case of subjective and relative clauses. Similarly, expressions for concepts which comprise places, etc., can be formed. Here too, it is to be remarked, the sense of the subordinate clauses cannot be rendered in a principal clause, because an essential constituent, namely the determination of place and time, is missing and only alluded to by a relative pronoun or a conjunction.¹⁰

In conditional clauses also, there is, just as we have realized in the case of subjective, relative and adverbial clauses, a constituent with indeterminate indication corresponding to which there is a similar one in the concluding clause. In referring to one another the two clauses combine into a whole which expresses, as a rule, only one proposition. In the sentence "if a number is smaller than 1 and greater than 0, then its square also is smaller than 1 and greater than 0" this constituent in the conditional clause is 'a number' and in the concluding clause it is 'its'. Just through this indeterminacy the sense acquires the universal character which one expects of a law. But it is in this way also that it comes about that the conditional clause alone does not possess a complete proposition as its sense, and that together with the concluding clause it expresses a single proposition whose parts are no longer propositions. It is not generally the case that a hypothetical judgment correlates two judgments. Putting it in that (or a similar) manner would amount to using the word 'judgment' in the same sense that I have attributed to the word 'proposition'. In that case I would have to say: in a hypothetical proposition two propositions are re-

¹⁰ Regarding these sentences, however, several interpretations are easily conceivable. The sense of the sentence "after Schleswig-Holstein was torn away from Denmark, Prussia and Austria fell out with one another" could also be rendered by "after the separation of Schl.-H. from Denmark, Prussia and Austria fell out with one another" In this formulation it is sufficiently clear that we should not regard it as part of this sense that Schleswig-Holstein once was separated from Denmark; but rather that this is the necessary presupposition for the very existence of a nominatum of the expression 'after the separation of Schl.-H. from D.' Yet, our sentence could also be interpreted to the effect that Schl.-H. was once separated from D. This case will be considered later. In order to grasp the difference more clearly, let us identify ourselves with the mind of a Chinese who, with his trifling knowledge of European history, regards it as false that Schl.-H. ever was separated from D. This Chinese will regard as neither true nor false the sentence as interpreted in the first manner. He would deny to it any nominatum because the dependent clause would be lacking a nominatum. The dependent clause would only apparently indicate a temporal determination. But if the Chinese interprets our sentence in the second manner, then he will find it expressing a proposition which he would consider false, in addition to a component which, for him, would be without nominatum.

lated to each other. But this could be the case only if an indeterminately denoting constituent were absent;¹¹ but then universality would also be missing.

If a time point is to be indeterminately indicated in a conditional and a concluding clause, then this is not infrequently effected by *tempus praesens* of the verb, which in this case does not connote the present time. It is this grammatical form which takes the place of the indeterminately indicating constituent in the main and the dependent clause. "When the sun is at the Tropic of Cancer, the northern hemisphere has its longest day" is an example. Here, too, it is impossible to express the sense of the dependent clause in a main clause. For this sense is not a complete proposition; if we said: "the sun is at the Tropic of Cancer" we would be referring to the present time and thereby alter the sense. Similarly, the sense of the main clause is not a proposition either, only the whole consisting of main and dependent clause contains a proposition. Further, it may occur that several constituents common to conditional and concluding clause are indeterminately indicated.

It is obvious that subjective clauses containing 'who', 'what', and adverbial clauses with 'where', 'when', 'wherever', 'whenever' are frequently to be interpreted, inasmuch as their sense is concerned, as conditional sentences; e. g., "He who touches pitch soils himself".

Conditional clauses can also be replaced by relative clauses. The sense of the previously mentioned sentence can also be rendered by "the square of a number which is smaller than 1 and larger than 0, is smaller than 1 and larger than 0."

Quite different is the case in which the common constituent of main and dependent clause is represented by a proper name. In the sentence: "Napoleon who recognized the danger to his right flank, personally led his troops against the enemy's position" there are expressed two propositions:

1. Napoleon recognized the danger to his right flank.
2. Napoleon personally led his troops against the enemy's position.

When and where this happened can indeed be known only from the context, but is to be viewed as thereby determined. If we pronounce our whole sentence as an assertion we thereby assert simultaneously its two component sentences. If one of the components is false the whole is false. Here we have a case in which the dependent clause by itself has a sense in a complete proposition (if supplemented by temporal and spatial indications). The nominatum of such a clause is therefore a truth-value. We may therefore expect that we can replace it by a sentence of the same truth value without altering the truth of the whole. This is indeed the case;

¹¹ Occasionally there is no explicit linguistic indication and the interpretation has to depend upon the total context.

but it must be kept in mind that for a purely grammatical reason, its subject must be 'Napoleon'; because only then can the sentence be rendered in the form of a relative clause attaching to 'Napoleon'. If the demand to render it in this form and if the conjunction with 'and' is admitted, then this limitation falls away.

Likewise, in dependent clauses with 'although' complete propositions are expressed. This conjunction really has no sense and does not affect the sense of the sentence; rather, it illuminates it in a peculiar fashion.¹² Without affecting the truth of the whole the implicate may be replaced by one of the same truth value; but the illumination might then easily appear inappropriate, just as if one were to sing a song of sad content in a cheerful manner.

In these last instances the truth of the whole implied the truth of the component sentences. The situation is different if a conditional sentence expresses a complete proposition; namely, when in doing so it contains instead of a merely indicating constituent a proper name or something deemed equivalent to a proper name. In the sentence: "if the sun has already risen by now, the sky is heavily overcast", the tense is the present—therefore determinate. The place also is to be considered determinate. Here we can say that a relation is posited such that the case does not arise in which the antecedent sentence nominates the True and the consequent sentence nominates the False. Accordingly, the given (whole) sentence is true if the sun has not as yet risen (no matter whether or no the sky be heavily overcast), and also if the sun has risen and the sky is heavily overcast. Since all that matters are only the truth-values, each of the component sentences can be replaced by another one of the same truth-value, without altering the truth-value of the whole sentence. In this case also, the illumination would usually seem inappropriate; the proposition could easily appear absurd; but this has nothing to do with the truth-value of the sentence. It must always be remembered that associated thoughts are evoked on the side; but these are not really expressed and must therefore not be taken account of; their truth-values cannot be relevant.¹³

We may hope we have considered the simple types of sentences. Let us now review what we have found out!

The sense of a subordinate clause is usually not a proposition but only part of one. Its nominatum is therefore not a truth-value. The reason for this is *either*: that the words in the subordinate clause have only indirect nominata, so that the nominatum, not the sense, of the clause is a proposition, *or*, that the clause, because of a contained indeterminately indicating constituent, is incomplete, such that only together with the principal

¹² Similarly in the cases of 'but', 'yet'.

¹³ The proposition of the sentence could also be formulated thus: "either the sun has not as yet risen or the sky is heavily overcast". This shows how to interpret this type of compound sentence.

clause does it express a proposition. However, there are also instances in which the sense of the dependent clause is a complete proposition, and in this case it can be replaced by another clause of the same truth-value without altering the truth-value of the whole; that is, inasmuch as there are no grammatical obstacles in the way.

In a survey of the various occurrent clauses one will readily encounter some which will not properly fit within any of the considered divisions. As far as I can see, the reason for that is that these clauses do not have quite so simple a sense. It seems that almost always we connect associated propositions with the main proposition which we express; these associated propositions, even if unexpressed, are associated with our words according to psychological laws also by the listener. And because they appear as associated automatically with our words (as in the case of the main proposition) we seem to wish, after all, to express such associated propositions along with the main propositions. The sense of the sentence thereby becomes richer and it may well happen that we may have more simple propositions than sentences. In some cases the sentence may be interpreted in this way, in others, it may be doubtful whether the associated proposition belongs to the sense of the sentence or whether it merely accompanies it.¹⁴ One might find that in the sentence: "Napoleon, who recognized the danger to his right flank, personally led his troops against the enemy's position" there are not only the previously specified two propositions, but also the proposition that the recognition of the danger was the reason why he led his troops against the enemy. One may indeed wonder whether this proposition is merely lightly suggested or actually expressed. Consider the question whether our sentence would be false if Napoleon's resolution had been formed before the recognition of the danger. If our sentence were true even despite this, then the associated proposition should not be regarded as part of the sense of the sentence. In the alternative case the situation is rather complicated: we should then have more simple propositions than sentences. Now if we replaced the sentence: "Napoleon recognized the danger for his right flank" by another sentence of the same truth-value, e. g., by: "Napoleon was over 45 years old" this would change not only our first but also our third proposition; and this might thereby change also the truth-value of the third proposition—namely, if his age was not the reason for his resolution to lead the troops against the enemy. Hence, it is clear that in such instances sentences of the same truth-value cannot always be substituted for one another. The sentence merely by virtue of its connection with another expresses something more than it would by itself alone.

Let us now consider cases in which this occurs regularly. In the sentence: "Bebel imagines that France's desire for vengeance could be assuaged

¹⁴ This may be of importance in the question as to whether a given assertion be a lie, an oath or a perjury.

by the restitution of Alsace-Lorraine" there are expressed two propositions, which, however, do not correspond to the main and the dependent clause—namely:

1. Bebel believes that France's desire for vengeance could be assuaged by the restitution of Alsace-Lorraine;
2. France's desire for vengeance cannot be assuaged by the restitution of Alsace-Lorraine.

In the expression of the first proposition the words of the dependent clause have indirect nominata; while the same words, in the expression of the second proposition, have their usual nominata. Hence, we see that the dependent clause of our original sentence really is to be interpreted in a twofold way; i. e., with different nominata, one of which is a proposition and the other a truth-value. An analogous situation prevails with expressions like 'to know', 'to recognize', 'it is known'.

A condition clause and its related main clause express several propositions which, however, do not correspond one-to-one to the clauses. The sentence: "Since ice is specifically lighter than water, it floats on water" asserts:

1. Ice is specifically lighter than water.
2. If something is specifically lighter than water, it floats on water.
3. Ice floats on water.

The third proposition, being implied by the first two, would perhaps not have to be mentioned expressly. However, neither the first and the third, nor the second and the third together would completely render the sense of our sentence. Thus we see that the dependent clause 'since ice is specifically lighter than water' expresses both our first proposition and part of the second. Hence, our clause cannot be replaced by another of the same truth-value; for thereby we are apt to alter our second proposition and could easily affect its truth-value.

A similar situation holds in the case of the sentence: "If iron were lighter than water it would float on water". Here we have the two propositions that iron is not lighter than water and that whatever is lighter than water floats on water. The clause again expresses the one proposition and part of the other. If we interpret the previously discussed sentence: "After Schleswig-Holstein was separated from Denmark, Prussia and Austria fell out with one another" as containing the proposition that Schleswig-Holstein once was separated from Denmark, then we have: firstly, this proposition, secondly, the proposition that, at a time more precisely determined by the dependent clause, Prussia and Austria fell out with one another. Here, too, the dependent clause expresses not only one proposition but also part of another. Therefore, it may not generally be replaced by another clause of the same truth-value.

It is difficult to exhaust all possibilities that present themselves in language; but I hope, in essence at least, to have disclosed the reasons why, in view of the invariance of the truth of a whole sentence, a clause cannot always be replaced by another of the same truth-value. These reasons are:

1. that the clause does not denote a truth-value in that it expresses only a part of a proposition;
2. that the clause, while it does denote a truth-value, is not restricted to this function in that its sense comprises, beside one proposition, also a part of another.

The first case holds

- a.* with the indirect nominata of the words;
- b.* if a part of the sentence indicates only indirectly without being a proper name.

In the second case the clause is to be interpreted in a twofold manner; namely, once with its usual nominatum; the other time with its indirect nominatum; or else, the sense of a part of the clause may simultaneously be a constituent of another proposition which, together with the sense expressed in the dependent clause, amounts to the total sense of the main and the dependent clause.

This makes it sufficiently plausible that instances in which a clause is not replaceable by another of the same truth-value do not disprove our view that the nominatum of a sentence is its truth-value and its sense a proposition.

Let us return to our point of departure now.

When we discerned generally a difference in cognitive significance between "a = a" and "a = b" then this is now explained by the fact that for the cognitive significance of a sentence the sense (the proposition expressed) is no less relevant than its nominatum (the truth-value). If a = b, then the nominatum of 'a' and of 'b' is indeed the same and therefore also the truth-value of "a = b" is the same as that of "a = a". Nevertheless, the sense of 'b' may differ from the sense of 'a'; and therefore the proposition expressed by "a = b" may differ from the proposition expressed by "a = a"; in that case the two sentences do not have the same cognitive significance. Thus, if, as above, we mean by 'judgment' the transition from a proposition to its truth-value, then we can also say that the judgments differ from one another.