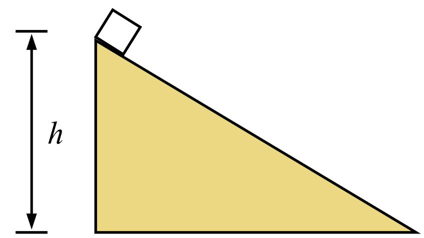
Chapter 2. Extra Problem, A Paradox.

X2.1 Consider the following standard problem and its "standard" solution:

A block on a friction-free incline is released from rest at height h above the bottom (see figure). Find the speed of the block at the bottom of the incline. Neglect air resistance.



Standard Solution (Lab frame)

Energy conservation says that $T_{Top} + U_{Top} = T_{Bottom} + U_{Bottom}$

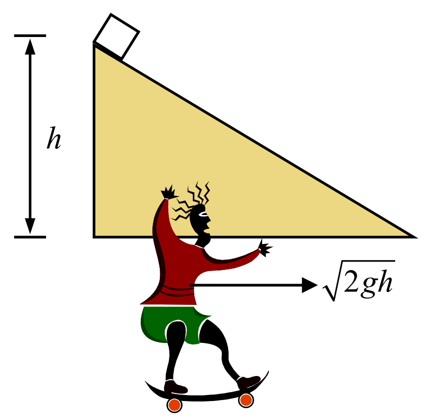
We take the reference point for the potential energy to be at the bottom, i.e. $U_{Bottom} = 0$. Since the block starts from rest, $T_{Top} = 0$. Therefore,

$$0 + mgh = \tfrac{1}{2} m\mathrm{v}_{Bottom}^2 \quad \rightarrow \quad \mathrm{v}_{Bottom} = \sqrt{2gh}\,.$$

Answer: The speed of the block at the bottom of the incline is $\sqrt{2gh}$. This is the same as the answer in Example 2.1 with $h = x_0 \sin\theta$.

Moving Frame Solution

Now suppose that you are an observer on a skateboard moving to the right with speed equal to the block's speed at the bottom. How would you solve the problem now?



Same strategy as before except that in this reference frame, the block is initially moving to the left and that at the bottom of the incline it must have zero speed.

Energy conservation says that $T_{Top} + U_{Top} = T_{Bottom} + U_{Bottom}$

We take the reference point for the potential energy to be at the bottom, i.e. $U_{Bottom} = 0$. In this frame, the block is moving to the left with speed $\mathrm{v} = \sqrt{2gh}$.

Then, $T_{Top} = \tfrac{1}{2} m\left(\sqrt{2gh}\right)^2 = mgh$. Therefore,

$$mgh + mgh = 0 + 0 \quad \rightarrow \quad 2mgh = 0 \quad ???$$

Either there is something wrong with one or both solutions, or mechanical energy in a moving inertial frame is not conserved. Find where the problem lies and provide a correct solution in each frame.