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On the Randomness of Pulsar Nulls

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ABSTRACT
Pulsar nulling is not always a random process; most pulsars, in fact, null non-randomly. The Wald-Wolfowitz statistical runs test is a simple diagnostic that pulsar astronomers can use to identify pulsars that have non-random nulls. It is not clear at this point how the dichotomy in pulsar nulling randomness is related to the underlying nulling phenomenon, but its nature suggests that there are at least two distinct reasons that pulsars null.

Key words: MHD — plasmas — polarization — radiation mechanisms: non-thermal — pulsars: general

1 INTRODUCTION

Pulsar nulling is the sudden cessation in pulsar emission, a phenomenon that remains largely unexplained since its discovery by Backer (1970). Nulls are most easily distinguished from the normal pulsar emission (hereafter, bursts) in histograms of the observed intensity of individual pulses, as seen in Figure 1. In this figure, the solid-line histogram is the distribution of the integrated pulsar intensity (normalized with respect to the average) when the pulsar's beam is pointing towards the Earth, while the dashed-line histogram indicates the same quantity when the pulsar's beam is pointed away from the Earth. Nulls are indicated by the population of intensity around $0 \times I/ < I >$, and bursts are located at higher intensities, centred near $1 \times I/ < I >$. The dotted vertical line indicates the value that best distinguishes nulls from pulses.

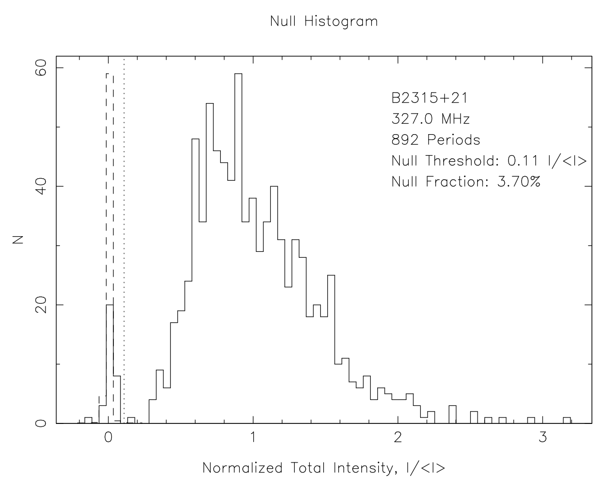

Figure 1. The null histogram of pulsar B2315+21. The solid histogram indicates the distribution of the normalized total intensity detected when the pulsar is emitting in the direction of the Earth, and the dashed-line histogram indicates the same quantity when the pulsar is pointed away from the Earth. Nulls have an energy distribution consistent with the background, around $0 \times I/ < I >$, while bursts are centred around $1 \times I/ < I >$. The vertical dotted line indicates the average intensity threshold chosen to distinguish nulls from bursts.

Following the classic studies by Ritchings (1976) & Biggs (1992), nulls have generally been regarded as *random* in occurrence and *cessations* of the pulsar-emission mechanism. Little observational evidence challenged these presumptions until recently, as a number of intriguing clues to the underlying physics have been uncovered. In an earlier study, Redman et al. (2005) showed that the nulls of B2303+30 occurred exclusively during one of the pulsar's two emission modes, which were distinguishable via the subpulse drift rate. The nulls of both B0834+06 and J1819+1305 were shown to exhibit periodicities related to the subpulse modulation (Rankin & Wright 2007, 2008); and fluctuation features produced by null periodicities have now been identified in a number of pulsars (Herfindal & Rankin 2007, 2008). The implication of these results, apparently, is that many nulls are the result of "empty" sightline traverses through a rotating "carousel" of emitting subbeams (Deshpande & Rankin 2001). Bhat et al. (2007) find intriguing evidence that the nulls of B1133+16 are not simultaneous across all frequency bands, suggesting that some nulls are not simply broadband cessations of pulsar emission. However, the remarkable timing study by Kramer et al. (2006) of B1931+24 indicates that this star's quasi-periodic, month-long nulls do represent a turn-off of its emission processes.

Clearly, we have much to learn about the nulling phenomenon. Therefore, we have sought to develop a further tool for such investigations and to apply it to a suitable population of pulsars. In § 2, we discuss our observations. In § 3, we review the Wald-Wolfowitz runs test, which we use to determine the randomness of the distribution of nulls and pulses in § 4. In § 5, we discuss our results, with special attention to B0834+06, before drawing our conclusions in § 6.

2 OBSERVATIONS

The observations were carried out using the 305-meter Arecibo Telescope in Puerto Rico. All of the observations used the upgraded instrument with its Gregorian feed system, 327-MHz (P band) receiver, and Wideband Arecibo Pulsar Processor (WAPP[1]). The ACFs and CCFs of the channel voltages produced by receivers connected to orthogonal linearly (circularly, after 2004 October 11) polarized feeds were 3-level sampled. Upon Fourier transforming, some 64 channels were synthesized across a 25-MHz bandpass with about a milliperiod sampling time. Each of the Stokes parameters were corrected for interstellar Faraday rotation, various instrumental polarization effects, and dispersion.

3 THE RUNS TEST

The runs test, also known as the *Wald-Wolfowitz runs test* (Wald & Wolfowitz 1940), is a statistical procedure for determining whether an observed binary sequence, such a series of coin tosses, supports the hypothesis that the order of the elements is random. The runs test has many variations; we have chosen to utilize the “number of runs” test. For a summary of some other variants, see Bradley (1968).

A *run* is an unbroken series of like terms. The randomness of a sequence depends upon the total number of heads (n_1) and tails (n_2) in the sequence, and the number of observed runs (R). When $N = n_1 + n_2$ is large (*i.e.,* greater than 20, with n_1 and n_2 each greater than or equal to 10), the distribution of values of R is approximately normal, with a mean of:

$$\mu = \frac{2n_1n_2}{n_1+n_2} + 1 \quad (1)$$

and a standard deviation given by:

$$\sigma = \sqrt{\frac{2n_1n_2(2n_1n_2 - n_1 - n_2)}{(n_1+n_2)^2(n_1+n_2-1)}} \quad (2)$$

such that the statistic

$$Z = \frac{R-\mu}{\sigma} \quad (3)$$

has an approximate standard normal distribution (*i.e.,* Gaussian distribution with mean 0 and standard deviation 1). The statistic Z represents how far, as a multiple of the standard deviation, the observed number of runs is located from the expected value.

Figure 2 shows a histogram of 8192 Z values for a Monte Carlo simulation of 1024 random pulse sequences, each with a null fraction (the fraction of time the pulsar is found in the null state) of 32% (chosen arbitrarily). The vertical dashed lines (at $Z = -1.96$ and $Z = 1.96$) indicate the confidence interval which, in theory, contains approximately 95% of the observed Z-values. In the figure, these bounds contain 94.91% of the observed Z values. For binary sequences, such as the pulsar bursts and nulls, $-1.96 < Z < 1.96$ indicates that the sequence is indistinguishable from a random process at the $\alpha = 0.05$ significance level, and the hypothesis of randomness cannot be rejected. Otherwise, the hypothesis of randomness is rejected. This result is independent of the null fraction. Note that there are two types of non-random sequences: those that are “over-clustered” ($Z > 1.96$) and those that are “over-scattered” ($Z < -1.96$).

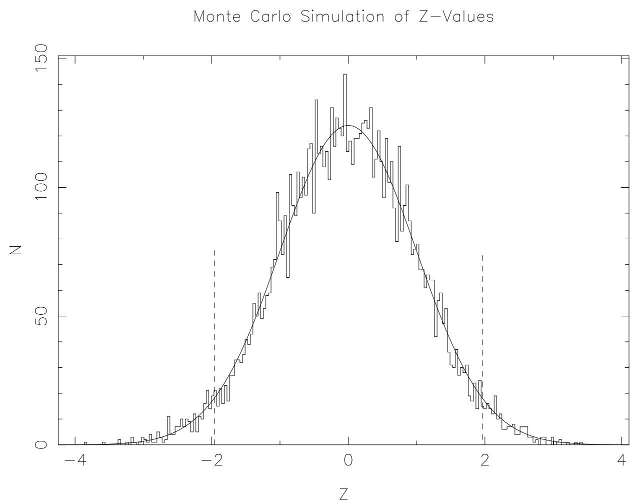

Figure 2. Monte Carlo simulations of 8192 1024-period sequences, where each pulse has a 32% change of being a null. The Gaussian indicates the expected distribution of Z-values. According to the runs test, there is a 5% chance that the observed value of Z will be more than 1.96σ (vertical dashed lines) from the mean value, which is corroborated here by our empirical measurement of $\alpha(= 0.0509)$, the fraction of observed sequences that fall outside this boundary. The average μ in this simulation was 446.12, close to the expected value of 446.64 (see Equation 1).

4 RESULTS

Before analyzing our many single-pulse sequences, we needed to carefully eliminate interference, which could bias our results (as the interference might be random or periodic). Most interference exists as spurious and unexpected increases in the measured intensity or polarization. We used Chauvenet’s criterion to identify single pulses with intensity and/or polarization measurements with a $< 0.5\%$ probability of being part of the rest of the observed sequence (assuming an underlying Gaussian distribution). Explicitly, we eliminated outliers that matched the following criterion:

$$N * P(Z) < 0.5\% \quad (4)$$

where N is the total number of single pulses in the sequence, Z is the number of standard deviations the value resides from the mean (identical to the Z defined in the previous section, except for a different population), and $P(Z)$ is the probability of observing a value $Z\sigma$ from μ. While total intensity histograms are certainly not Gaussians (e.g., Fig. 1),

[1] http://www.naic.edu/∼wapp

Table 1. Pulsar Nulling runs test Statistics

Pulsar Name	Null Fraction	Reject Randomness? Yes	No	Average Z $\pm\sigma_Z$
B0045+33	21%	3	0	-3.5 ± 0.9
B0301+19	13%	15	0	-16.3 ± 6.4
B0525+21	25%	11	0	-8.0 ± 1.5
B0751+32	39%	16	0	-7.6 ± 3.4
B0823+26	7%	8	5	-2.6 ± 1.5
B0834+06	9%	1	6	0.2 ± 1.2
B1133+16	20%	7	2	-3.5 ± 1.7
B1612+07	10%	0	4	-0.5 ± 1.4
B1848+12	54%	7	0	-12.2 ± 6.7
B1942–00	28%	3	0	-7.8 ± 0.4
B2110+27	30%	4	10	-1.1 ± 1.4
B2122+13	22%	13	2	-3.1 ± 1.0
B2303+30	11%	5	0	-6.5 ± 2.1
B2315+21	3%	0	4	-0.8 ± 0.5
J0540+32	53%	3	2	-2.5 ± 1.9
J1649+2533	20%	5	0	-9.7 ± 3.0
J1752+2359	81%	2	1	-6.3 ± 7.9
J2253+1516	49%	4	0	-12.3 ± 3.8

this criterion is conservative compared to the interference we observe in our pulse sequences.

The removal of individual "pulses" from the single-pulse sequence cuts the observed sequence into several shorter sequences. Each smaller pulse sequence provides us with an independent, statistically-valid runs test, as long as the sequence contains a minimum of ten nulls and ten pulses. In practice, while most observations exhibited some interference, these sequences were very clean.

Our results are presented in Table 1. For each pulsar, we provide the fraction of time the pulsar was observed in the null state, the number of sequences for which we did ($|Z| > 1.96$) or did not reject randomness, and the average value of Z amongst all sequences, along with the standard deviation in the measured value of Z.

5 DISCUSSION

The majority of pulsars in our sample null non-randomly, but at least three of them null at intervals which are consistent with a random process: B0834+06, B1612+07 and B2315+21. Furthermore, it is clear that, of the pulsars that null non-randomly, all do so with a bias towards "over-clustering" ($Z < -1.96$) — that is, that non-random nulls occur in groups.

There are at least two possible explanations for this behavioral dichotomy. First, there may be more than one reason that pulsar emission temporarily ceases (for example, some nulls may be due to absolute cessations of emission from the pulsar, while others may be produced by "empty" sightline passes through the subbeam structure of the emission). Second, nulls may always be random, but the conditions required for nulling may be non-random. We see a phenomenon of this nature in B2303+30, where nulls occur mostly (and perhaps exclusively) in one of the subbeam drift modes (the "Q mode"). However, a close examination of long Q-mode sequences of B2303+30 confirmed the non-random nature of the nulls within those sequences. Therefore, we are inclined to support the hypothesis that there are multiple reasons that pulsars null, some of which are indistinguishable from random processes.

The selection of these pulsars was not unbiased. Overlap between the burst and null populations (most easily seen in null histograms) biases the sequence towards randomness. Therefore, we specifically chose pulsars that had well-defined null and pulse populations. As our ability to measure the intensity of fainter pulsars improves, the population of pulsars to which we can apply the runs test will grow.

The nulling behavior of several of these pulsars has already been mentioned in the literature. In addition to the aforementioned publications, Lewandowski et al. (2004) noted that pulsar J1752+2359 exhibits a predictable pulse emission decay behavior. Weltevrede et al. (2006) suggested that there was a relationship between the long-period feature of B1133+16 and its nulls, as well as a possible distortion in the drift bands of B2110+27 due to nulling. We also note that J1752+2359 has been observed to exhibit giant pulses (Ershov & Kuzmin 2006). The runs test provides pulsar astronomers with a quick and easy way to potentially identify other similar emission oddities.

5.1 Burst-length histograms

The runs test for B0834+06 from our sample appears to contradict earlier results (Rankin & Wright 2007). In that paper, the authors used a "burst-length histogram" to estimate the randomness of uninterrupted pulse sequences of length x. This is another modification of the runs test, with each burst-length acting as an independent (albeit less-accurate) sample of the randomness of the sequence.

The distribution of the expected values of a burst-length histogram for a random sequence of pulses and nulls is the probability of observing a null followed by x bursts, followed by another null. Thus, the expected values are:

$$N_\mu(x) = N f_n^2 f_b^x \qquad (5)$$

where N is the total number of periods in the sequence, f_n is the null fraction, and $f_b = 1 - f_n$ is the burst fraction. The standard deviation in each burst length is $\sqrt{N_\mu(x)}$. The same logic can be used to calculate the expected distribution of the null-length histogram, with analogous equations (exchanging the burst and null fractions).

Finding deviations from the expected value in a burst-length histogram is complicated by the number of bins in the histogram, as each is an independent test for randomness. Thus the probability of finding a deviation *somewhere* in the burst-length histogram is increased — the number of false positives in a burst-length histogram is related to the length of the sequence, and inversely related to the null fraction. Therefore, the technique should only be used if you have multiple sequences to examine and if the deviations are strongly non-random.

B0834+06 meets both the latter conditions. Figure 3 shows the burst-length histogram from an observation taken in 2003. Monte Carlo simulations indicate that a sequence such as this (with a null fraction of 9.13% and sequence length of 3140 periods) will exhibit 4.0 false positives, on average. This burst length histogram has 7 burst lengths outside the range expected for a random distribution at the $\alpha \leqslant 0.05$ significance level. Most of these deviations are quite small and not very significant, but the burst length of

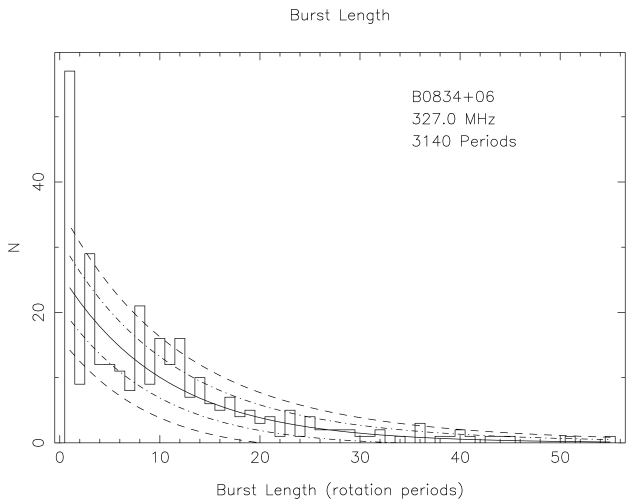

Figure 3. The updated burst-length histogram from Rankin & Wright (2007). The solid curve indicates the average expected value for a random distribution, the dash-dotted curve indicates the 1σ range, and the dashed lines contain the burst lengths that are within the expected values for the hypothesis of randomness at the $\pm 1.96\sigma$, $\alpha = 0.05$ level. Here, burst lengths of 1, 2, 3, 8, 12, 36, and 55 fall outside the latter of these bounds and may indicate non-random behavior. However, only the burst length of 1 is consistently outside the expected range among most of our observations. In this observation, the single-burst length is 6.8σ from the expected value.

1 is 6.8σ from its expected value, and this observation is not a unique case. Among the 5 observations we analyzed, this pulsar exhibits an unusual number of single burst pulses in 4 observations, as much as 7.7σ from the expected value, even though all but 3 of the 27 sub-sequences indicated that the total number of runs was indistinguishable from random. Therefore, we recommend that both tests be applied to single-pulse sequences, when appropriate.

6 CONCLUSIONS

The runs test is a relatively simple diagnostic procedure for testing whether a binary sequence is random. We have shown how this test provides a valuable means for single-pulse pulsar astronomers to identify potentially interesting nulling pulsars. We have also quantified the analysis of burst-length histograms, with the warning that false indications of non-randomness should be expected and should be treated with skepticism, but may reveal insights into the non-randomness of nulls of specific lengths.

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