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Comparitive Study of Profile Stabilization in Slow, Fast and Millisecond Pulsars

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ABSTRACT
Correlation methods are used to examine the profile stabilization of both normal and millisecond pulsars. A number of the pulsars stabilized in a characteristic manner that shows a "parabolic" dependence—rationalizing earlier findings that showed power-laws with two different slopes. For the remaining objects, the distorting effects of mode changes or other profile dynamics are apparent in the correlations.

Key words: miscellaneous – profile stabilization – pulsar timing – methods: correlations, data analysis – pulsars: general, survey, millisecond

1 INTRODUCTION

This paper reports efforts over the last two years to measure and characterize the average profile stabilization characteristics for a group of pulsars that includes both well studied slow objects as well as some faster and even a few millisecond pulsars. For this analysis, we have attempted to reproduce the methods developed by Helfand *et al.* (1975; hereafter HMTIII) and later employed by Rathnasree & Rankin (1995; hereafter RR95) for groups of slow pulsars—although neither paper gives a precise description of its technique. Both, however, analyze the stabilization through use of cross-correlation functions (hereafter CCF) between a pulsar's global average profile and subaverage profiles computed from varying numbers of its pulses. The values of the average correlation coefficient as a function of subaverage profile length then provide a "snapshot" showing how a given star's profile stabilizes with integration length. Tracing how progressively longer subaverages consistently approach, or diverge from, the form of the global average profile quantifies the number of pulses necessary for building a stable average profile. An average profile comprised of such a number of pulses constitutes a reliable signature and encompasses a star's behavior over a specific timescale. In practical terms, knowledge of stabilization rates facilitates optimal timing strategies with minimal timing residuals in order, for example, to determine binary orbits or attempt the detection of gravitational waves. We note with interest the parallel work of Liu *et al.* (2011, 2012).

Following Helfand *et al.*, we then compute sets of average correlation coefficients χ_n for groups of both slow and millisecond pulsars by calculating the CCFs of each respective subaverage profile of n pulses with the global average profile. We then plot the quantities $1-\chi_n$ versus n on a logarithmic scale in order to depict how a pulsar's profile converges to a stable signature as the number of pulses used to construct the average increases. The minimum value of n for which χ_n is statistically significant is then interpreted as the most efficient integration length required for computing a stable average profile for a particular star.

Radio pulsars emit sequences of individual pulses—each one unique from the others and some nulled or "giant" as well as corrupted by radio-frequency interference—which can then be averaged synchronously to provide high signal-to-noise-ratio (hereafter S/N) profiles that exhibit remarkable stability over long periods of time. The four panels of Figure 1 exemplify how it is that the widely varied forms of single pulses, which almost never resemble a pulsar's global average profile, nonetheless aggregate to it progressively. The example

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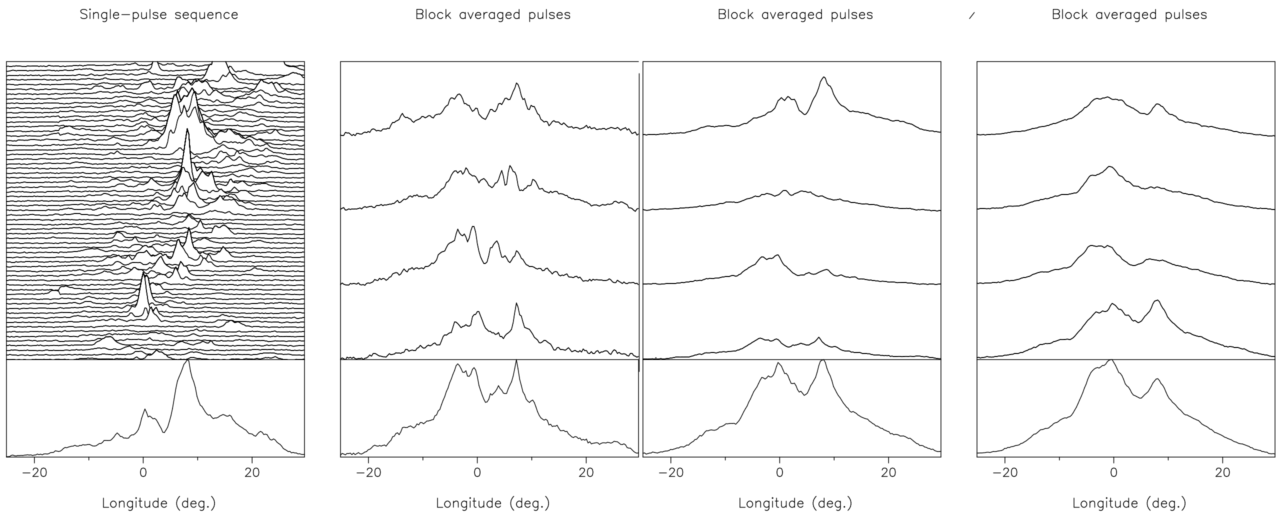

Figure 1. Profile formation in pulsar J1740+1000: a) (left display) shows 64 single pulses (main panel) together with their average subprofile (lower panel); b) (left central display) 64-pulse subprofiles together with their 256-pulse subprofile; c) (right central display) 256-pulse subprofiles together with their 1024-pulse average profile; and d) (right display) four nearly 1024-pulse subprofiles together with the total average profile.

Table 1. The eighteen observations used in the correlation analyses.

Pulsar	MJD	Frequency Band (MHz)	Pulsar Period (s)	S (mJy)	Number of Pulses	χ_0
J0437–4715	52004	1375	0.0058	142	9738	0.27 ± 0.13
J0631+1036	54540	1520	0.2878	0.8	3372	0.48 ± 0.17
B0656+14	52840	327	0.3849	6.5	24765	0.22 ± 0.19
B0656+14	53489	1525	0.3849	3.7	19107	0.28 ± 0.21
J0815+0939	54782	327	0.6451	3.7	3720	0.54 ± 0.14
B0950+08	53703	327	0.2530	400	23321	0.72 ± 0.19
B1133+16	53703	327	1.1878	257	2695	0.65 ± 0.31
B1237+25	53378	327	1.3823	110	5209	0.72 ± 0.20
B1534+12	55637	1169	0.0379	36	15835	0.24 ± 0.19
J1713+0747	55632	1178	0.0046	8	65646	0.61 ± 0.18
J1740+1000	54541	1470	0.1541	9.2	3894	0.50 ± 0.13
B1822–09	55014	325	0.7690	36	37399	0.51 ± 0.14
B1855+09	55637	1178	0.0054	5	111910	0.58 ± 0.17
B1913+16	55637	1491	0.0591	0.9	15233	0.19 ± 0.27
B1929+10	53186	327	0.2265	303	18835	0.94 ± 0.07
B1930+22	54540	1400	0.1445	1.2	4151	0.82 ± 0.09
B1933+16	54540	1520	0.3587	42	3352	0.98 ± 0.02
B1944+17	53966	327	0.4406	40	7038	0.23 ± 0.37

given here is for PSR J1740+1000, but this process obtains for the vast majority of stars. Notice that the block averages of pulses, as the averaged length increases, more closely resembles the global average profile. The observation in the figure represents a span of 4096 pulses, yet none of those exhibit profile forms close to that of the average profile, shown in the bottom panels of each set of subaverages.

We can see in Fig. 1 that although the longer subaverage profiles bear more and more semblance to the global average—*e.g.*, between the 32-pulse and the 1024-pulse subaverages—the number of pulses necessary to produce a subprofile/global average correlation that is statistically favorable for stability may not follow in a linear fashion. Indeed, it has been suggested that the stability curve may follow power laws, but this premise is tested strenuously only when the individual analyses for several stars show convergence at both faster and slower rates. Our goal is

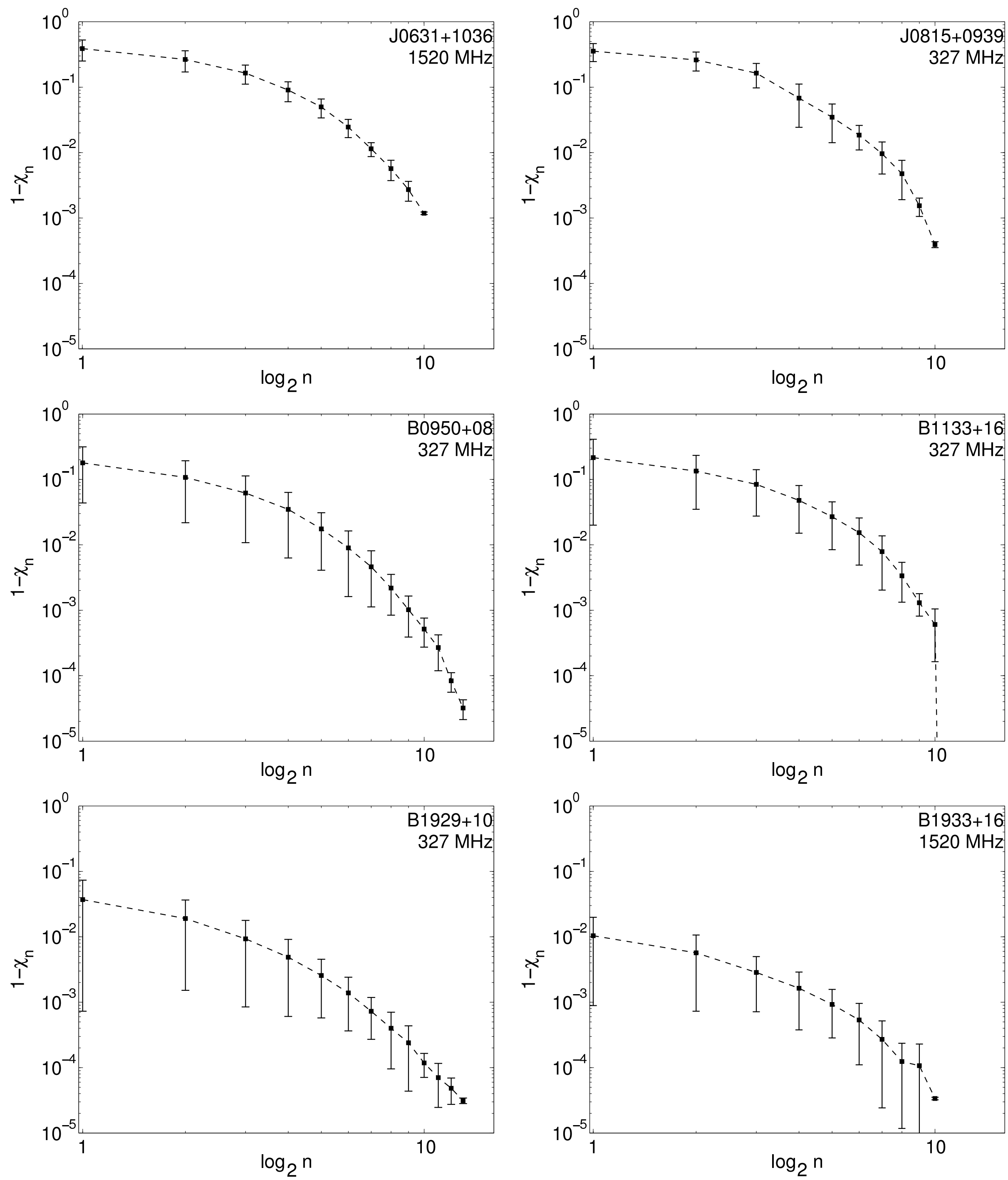

Figure 2. Correlation analysis results for the six "normal" pulsars in Table 1. Here $\log_{10}(1-\chi_n)$ is plotted against $\log_2 n$. The single-pulse correlation 2^0 is not plotted, but appears with its error in Table 1.

to portray the cross-correlation coefficient as a means of quantifying the subaverage resemblance to the global average, such that one may find a number n of subaveraged pulses for which the profile may be considered stable.

Here, a primary motivation is that of including millisecond pulsars (hereafter MSPs) in our stability analysis for the first time. On the one hand, major efforts are in progress to construct pulsar timing arrays in order

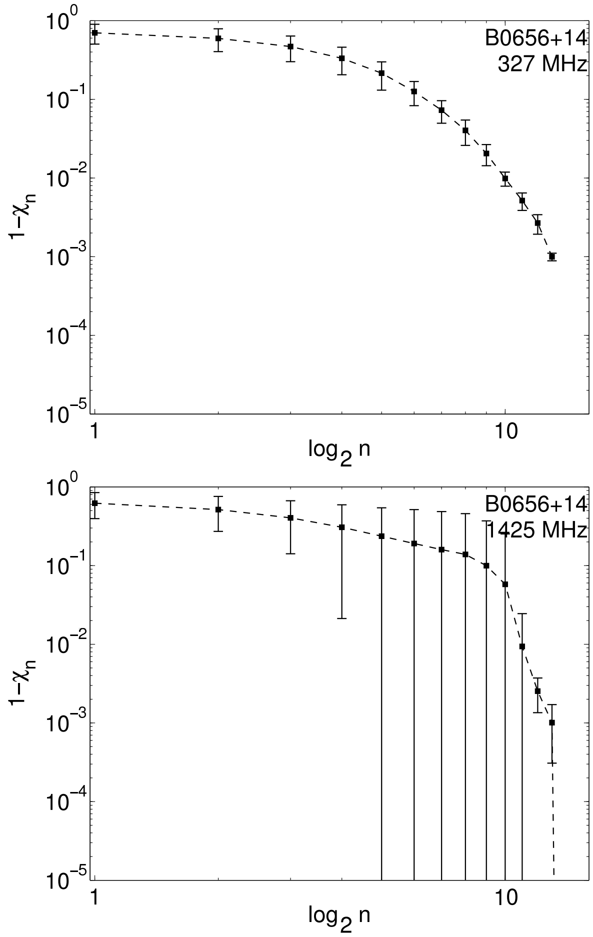

Figure 3. Correlation analysis results for the respective P and L-band observations of pulsar B0656+14 in Table 1 as in Fig. 2. Note the dramatic difference in profile dynamics.

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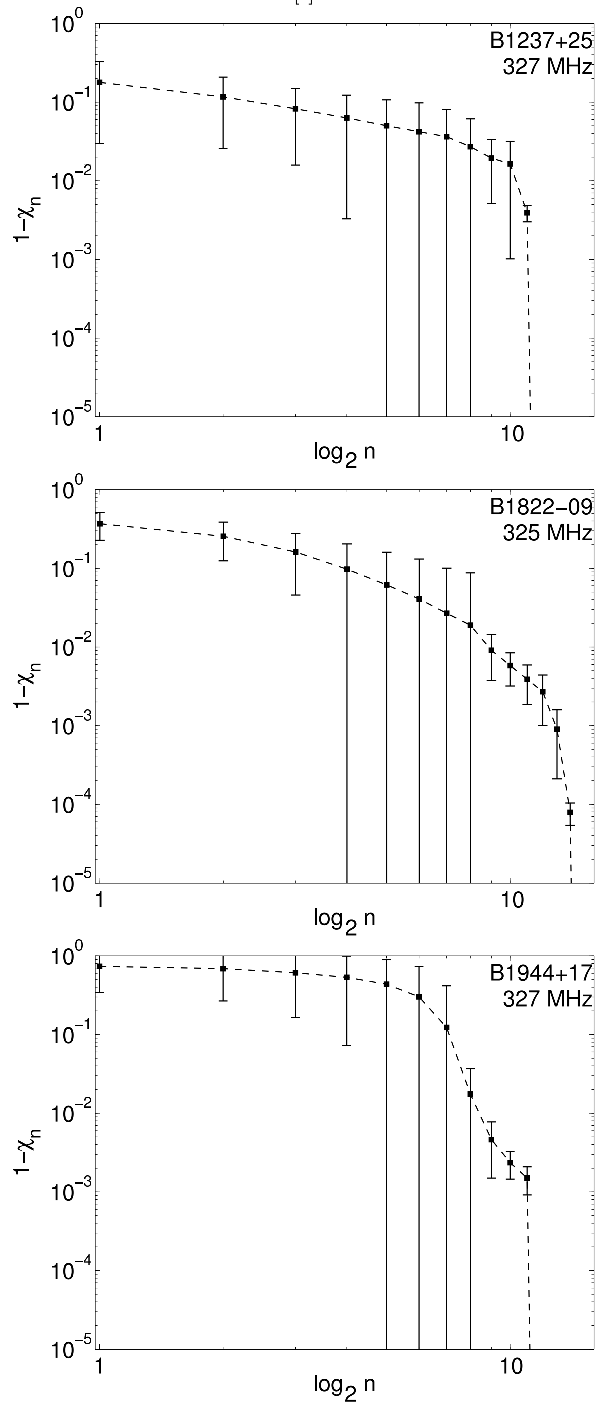


Figure 4. Correlation analysis results for the known mode-changing pulsars in Table 1 as in Fig. 2.

to search for gravitational waves (Foster & Backer 1990), and this requires further improvement of millisecond pulsar timing techniques. Specifically, in order to identify appropriate stars for membership in such arrays, criteria are needed to determine how well a particular star behaves as a cosmic clock, and a part of such assessment relates to profile stability. In short, we need measures both of the degree of stability and the rapidity of achieving it as both are relevant to clock accuracy and its determination.

Paradoxically, we often know little about the individual pulse behavior of millisecond pulsars because the S/N is degraded simply by the necessity to sample them so rapidly. On the other hand, then, this effort offers some promise to explore whether faster stars also exhibit the nulling, moding and drifting behaviors often encountered in their slower counterparts. Such dynamic emission features have been largely overlooked thus far in the process of pulsar timing, especially in the timing of MSPs. Of course, the motivation for ignoring dynamical effects in timing efforts stems from the appearance that the average

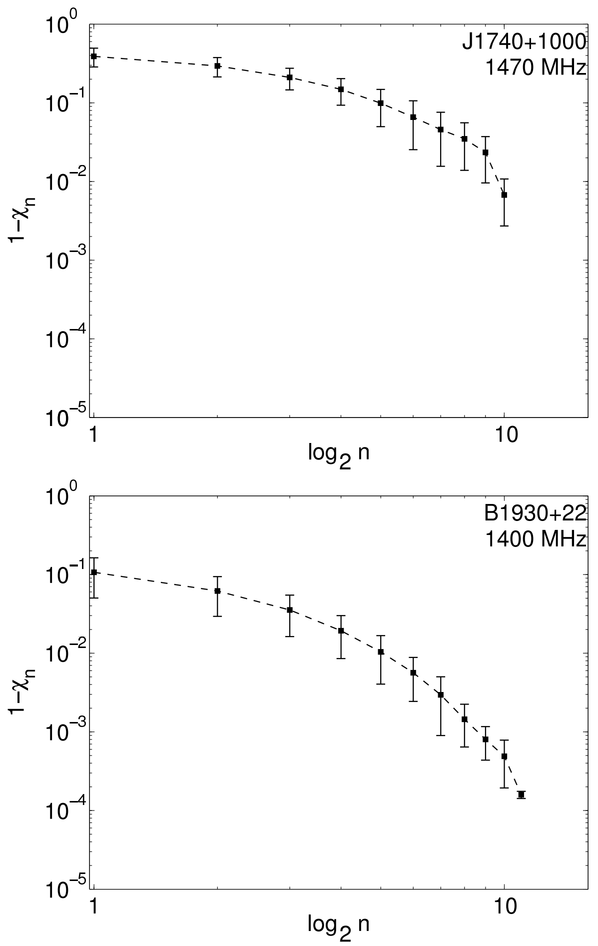


Figure 5. Correlation analysis results for the two fast pulsar observations in Table 1 as in Fig. 2.

profiles of bright, normal pulsars remain stable enough to this end, though even here there is little systematic quantitative evidence to support this assumption.

Finally, the profile stabilization analysis is also pertinent to studying dynamic behaviors in "normal" pulsars, particularly because many such pulsars may also be analyzed on a single pulse basis, allowing for a consistency test between the correlation models and what one actually observes in the pulse sequences of such stars. Together, experience applying the correlation method to both normal pulsars and millisecond pulsars provides a more useful tool for assessing profile stability and a more powerful means of identifying dynamic behaviors in weak pulse sequences.

2 PROFILE CORRELATION ANALYSIS

The convergence rate of the CCF coefficient χ_n, computed from sub- and global average profiles of the total power (*i.e.*, Stokes parameter I), is the primary indicator of single pulse to average-profile stabilization times. Given a global average profile of length G and subaverage profiles of length n, S_G and S_n respectively, the mean coefficient χ_n for cross correlations is given by

$$\chi_n = \frac{1}{N}\sum_{\alpha=1}^{N}\frac{\sum_i S_{nia}S_{Gi}}{\sqrt{\sum_i S_{nia}^2\sum_i S_{Gi}^2}} \qquad (1)$$

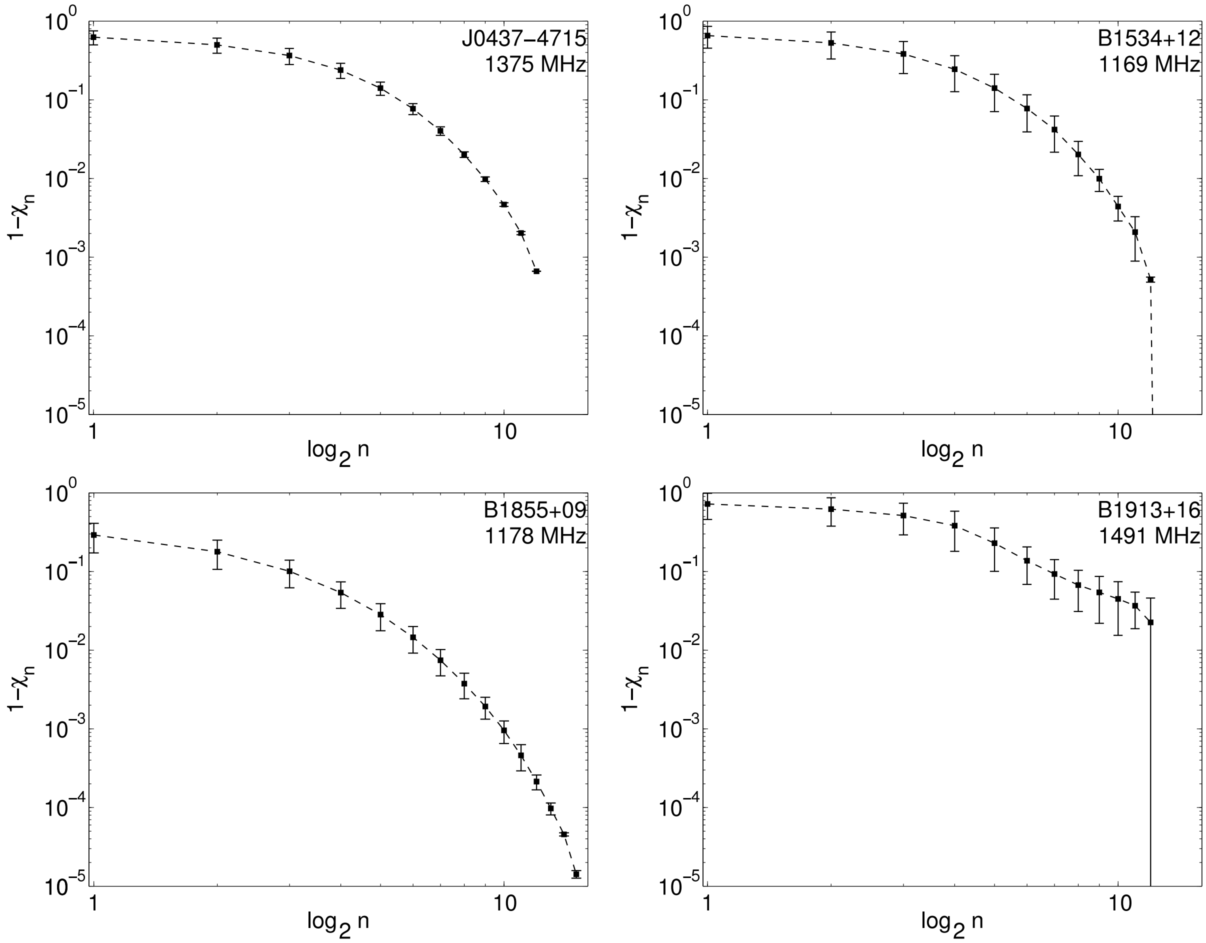
where $N = G/n$ modulo 2, and i is the bin index for the observation. As previously explained, the power-law decay exhibited by the plot of $(1-\chi_n)$ versus n tends to show distinct breaks in the slope for some pulsars, which often have coincided with the timescales of the aforementioned dynamic properties like drifting subpulses, moding, and nulling—but also with the overall complexity of the average profile (the intuitive reasoning suggesting that the more complex the profile, the slower or shallower the stabilization curve).

Both HMTIII and RR95 have shown that the stabilization rate of an individual pulsar corresponds not only with the presence of emission properties (*e.g.*, moding), but also to the time scales over which such properties manifest themselves. Even for stable, well studied pulsars, the stability curve has often seemed to follow power laws with two distinct slopes—a relatively shallow traverse at first and then a visibly steeper secondary slope at a characteristic integration length. Because these curves are of the form $(1-\chi_n)$ versus n, the shallow slope occurs at n values for which the subaverages take longer to stabilize; consequently, the steeper slope at larger n values represents a break in the prior slope, at which point the pulsar's profile stabilizes more rapidly, the quantity $(1-\chi_n)$ approaching zero as the correlation coefficient tends to unity.

Not only that!—the timescale at which these slope breaks occur also coincides with timescales of dynamic effects like the drifting of subpulses and moding. A stabilization curve may in fact display more than one break in its slope if the particular star exhibits behaviors at different time scales. If such an analysis, carried out on a large enough population of slow and millisecond pulsars, reveals mode-changing and other emission properties in even a few of these stars, such knowledge would allow for better initial estimates and more accurate and precise construction of average profiles for timing applications.

3 OBSERVATIONS AND RESULTS

Altogether, eighteen observations of seventeen stars were used in the correlation analyses. All of the observations, with the exception of pulsars J0437–4715 and B1822–09, were made using the 305-m Arecibo Observatory in Puerto Rico, at both 327 MHz and 1400 MHz (P- and L-bands, respectively). The observation of the millisecond pulsar J0437–4715 was conducted at 1375 MHz using the 64-m radio telescope at the Parkes Observatory in New



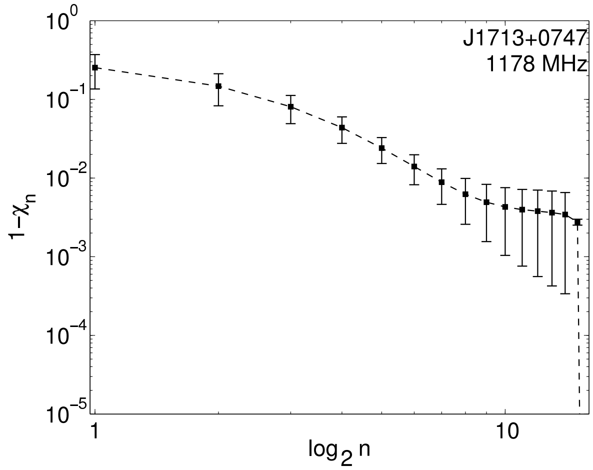


Figure 6. Correlation analysis results for the five MSPs in Table 1 as in Fig. 2.

South Wales, Australia. The observation used in the analysis of B1822–09 was made at 325 MHz by means of the Giant Metrewave Radio Telescope (GMRT) near Pune, India and is identical to that in Latham *et al.* (2012). The central frequency given on the correlation plots may not correspond exactly with the overall central frequency of each observation due to the fact that for some correlations specific bands were selected for quality reasons. Table 1 lists each observation with their relevant details.

4 DISCUSSION

The plots in Fig. 1 show the results of progressive profile averaging as applied to pulsar J1740+1000. The algorithm outlined in Section 2 was applied to the 17 pulsars in Table 1, consisting of eighteen total observations—accounting for both the P- and L-band information for PSR B0656+14. The error bars correspond to one standard deviation of the mean correlation coefficient χ, with errors extending beyond the plotted image due to the mathematical issue that in some correlations the sum of $(1-\chi)-\sigma \leqslant 0$ implies that $\chi+\sigma \geqslant 1$. The awkwardness of defining these values on a logarithmic scale result in the blown up errors, which simply correspond to standard deviation values approaching the unit boundary. That the mean correlation coefficient χ deviates outside of unity is neither physically nor mathematically meaningful, and results from the statistical presumption that χ may be Gaussian-distributed, when in fact it very likely represents a truncated Gaussian-distributed variable.

In the following sections we present our analysis results for groups of pulsars having either common pulse-sequence properties or similar stabilization characteristics.

4.1 "Normal" Pulsars

A subset of the slower pulsars in Table 1 exhibited a similar stabilization behavior, and the stabilization plots corresponding to six of these stars are shown in Figure 2. Note that in these plots the $\log_{10}(1-\chi_n)$ is plotted against $\log_2 n$, were n is partial profile length. Several common aspects of these plots are noteworthy. First, the overall stabilization curve shows a consistent "parabolic" dependence, such that for short subaverage profiles the stabilization is slow, but then becomes much more rapid for longer partial profile lengths. Of course, this general behavior is not surprising, but the "parabolic" character of the curves is seen clearly here for the first time. Earlier work tended to see the stabilization behavior as consisting of two power-law regions with different slopes rather than a progressive steepening. Second, the two-pulse partial profiles ($\log_2 1$) vary significantly in their values of χ_n—and extrapolating back to the single-pulse correlations (see Table 1), we see that pulsars vary considerably in the degree to which their single pulses resemble their average profiles. Clearly, the SPs of B1929+10 and B1933+16 highly resemble their respective average profiles with many pulses, the other four much less so. Finally, third, notice that the stabilization curves also resemble each other in terms of the amount of stabilization a particular averaging length gives. Or to be more concrete, one can see that the 1024-pulse averages ($\log_2 10$) result in $1-\chi_n$ values about three orders of magnitude smaller than the 2-pulse values.

This regular "parabolic" dependence can perhaps be easily understood. That the $\log_{10}[(1-\chi_n)/(1-\chi_0)]$ is roughly proportional to $1/(\log_2 n)^2$ implies that $1-\chi_n = (1-\chi_0)/n^{2c}$, where c is a constant of order unity. Then, referring back to eq.(1) one can see that just this sort of behavior will be produced by the first sum in the denominator (the second is a constant) such that the progressively longer sums will be averaged quadratically (as variances), rather than linearly as in the overall summation. This "parabolic" dependence then appears to reflect the usual increase in stability of sums as the *rms* variations of all "noise" sources are reduced through the aggregation of their variances. So, to the extent that this is so, this "parabolic" dependence is the fastest stabilization that can occur—and it suggests that all the sources of "noise" are highly random in nature.

4.2 B0656+14 at high and low frequency

Pulsar B0656+14 is well known for its "spiky" emission (Weltevrede *et al.* 2006a) and "Local RRAT" character. It is a strange pulsar with a difficult to understand profile and a small population of individual pulses so strong that partial profiles of a few hundred pulses vary greatly, depending on the properties of the one or more very bright single pulses within them (see Weltevrede *et al.* 2006a,b). Figure 3 shows contrasting stabilization curves for both a 327- (P band) and a 1425-MHz (L band) observation. Note the strong contrast between the two stabilization curves. The former exhibits a "textbook" example of the regular "parabolic" stabilization behavior discussed above; whereas the latter stabilizes very slowly at first and then more strongly only with integration lengths of several thousand pulses. We were surprised that any observation of this pulsar would exhibit the regular behavior of the former observations—and it is very likely that another such observation of this pulsar would not! Indeed, we seem to be learning here that the pulsar's "spiky" emission may not always be present.

4.3 Mode-Switching Pulsars

In order to study their stabilization concretely, we included observations of three well known mode-switching pulsars in our analysis. Figure 4 shows the results for pulsars B1237+25, B1822–09 and B1944+17, respectively. We see that B1237+25 stabilizes slowly in this observation up to an integration length of about 1000 pulses and more rapidly after that, but our other available observations of this pulsar show very different amounts and lengths of abnormal mode activity (*e.g.*, Srostlik & Rankin 2005; Smith & Rankin 2012), so each would probably have a different stabilization curve. The stabilization exhibited by pulsar B1822–09 in the very long 37,400-pulse observation (Latham *et al.* 2012) shows a mixture of behaviors: on short integration times, it shows the regular "parabolic" dependence, but on longer scales it stabilizes more slowly and exhibits clear breaks in slope at integration lengths of about 500 and 4000 pulses, respectively. Finally, pulsar B1944+17 stabilizes very slowly at shorter integration lengths, then strongly in averages of 500 or so, and then again more slowly for very long integrations. This is a pulsar with four modes and 2/3 null pulses (Kloumann & Rankin 2010), so again different observations would doubtlessly show different stabilization characteristics.

4.4 Fast Pulsars

We were able to study two fast pulsars, J1740+1000 and B1930+22, both with periods of some 150 ms. On the one hand these pulsars are interesting for our present purpose because their profiles are difficult to understand in geometric terms, and on the other they provide opportunity to study the stabilization of pulsars which have single-pulse S/N of only about unity. The results of our analyses are shown in Figure 5, and both stars seem to largely follow the regular "parabolic" behavior, though J1740+1000 does seem to show a break in slope around 500 pulses. Note also that B1930+22 exhibits single pulses that largely resemble its average profile.

4.5 Millisecond Pulsars

Stabilization analysis results for the five MSPs are shown in Figure 6, and we can see two strongly different behaviors: On the one had, pulsars J0437–4715, B1534+12 and B1855+09 all exhibit the strong "parabolic" stabilization behavior. The first two show SPs rather different than their long average profiles; whereas B1855+09's SPs more resemble its average profiles. Pulsar J1713+0747, however, stabilizes rather slowly in longer integrations up to some 16,000 pulses, despite the fact that its SPs well resemble its average profile. Finally, the Binary Pulsar B1913+16 exhibits a very slow stabilization in short integrations and hardly a strong stabilization even in averages of several thousand pulses. Clearly, it would be very interesting to study these MSPs at much longer integration lengths—as rather short times are involved—but such study remains costly in computer processing time.

5 DISCUSSION

In summary, our results are as follows—

- A number of slow and faster pulsars stabilize in a regular "parabolic" manner suggesting that the various sources of "noise" in the individual pulse stream can be regarded as a stationary statistical process.
- This regular process appears to reflect the manner in which the "noise" contributions to the correlation functions averages linearly over shorter scales and quadratically over longer ones.
- The single-pulse correlation functions χ_0 (see Table 1) provide a measure of the extent to which the individual pulses resemble the average profile, but this quantity appears to have little effect on the long term stabilization of a pulsar's profile.

Returning to HTMIII and RR95, our results above appear difficult to compare with these earlier studies. Neither has many objects in common with ours, and in both cases stabilization curves are presented for only a few objects. Overall, however, both studies suggest that the stabilization can often be described as a power law, possibly with regions of two different slopes; whereas that is not the behavior we find using the definition of χ_n in eq.(1). Therefore, we suspect that these earlier studies used a somewhat different technique, though (as mentioned above) neither gives an adequate description of their method.

Liu *et al.* (2012) do define their correlation (their eq. 1) in terms of the *difference* between short averages (or single pulses) and the global average, and their their stabilization curves indeed do show an overall power-law dependence. However, only for two MSPs, J0437–4715 and J1713+0747, can a direct comparison be made between the two methods. Even this is difficult, though, because for the former they have a PS of nearly 10^6 pulses available, and their shortest correlations are longer than our 4000 pulses. Nonetheless, their curve for this pulsar is a steep power law of nearly constant slope. By contrast, their curve for J1713+0747 is somewhat less steep everywhere up to $3{\cdot}10^5$ pulses and seems to show significant "breaks" for shorter averages of different lengths.

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