Chapter 21

Worlds Within Worlds—The Nucleus and Beyond

The exercises in this set emphasize material from the central sections of the chapter, with particular attention given to nuclear stability, binding energy, decay pathways, and kinetics. Table C-11 (stable isotopes) and Table C-12 (radioactive isotopes) provide supporting data. See Appendix C in PoC, *pages A77–A80.*

First-order kinetics, applicable to radioactive decay, is also covered in Chapter 18 (pages 653–656, Examples 18-4 and 18-5, and Exercises 21 through 27). Simple descriptions of nuclear structure and interactions appear in the Introduction (pages 6–7), Chapter 1 (page 23 and Example 1-5), and Chapter 2 (pages 46–50 and R2.2–R2.3, plus Example 2-2). For problems similar to the first two below, see also Exercises 5, 6, 9, and 10 in Chapter 2.

1. Determine the atomic number of the element (Z, the number of protons) by locating the appropriate chemical symbol in the periodic table. Then, from the superscript in each isotopic symbol ^{A}X, identify the *mass number A* (the combined number of protons and neutrons). Finally, solve for the *neutron number*

$$N = A - Z$$

by simple rearrangement of the equation

$$Z + N = A$$

Values of Z and N are collected in the table that follows:

	Element	Isotope	Z	N	A
(a)	boron	^{10}B	5	5	10
		^{11}B	5	6	11
		^{12}B	5	7	12
(b)	carbon	^{12}C	6	6	12
		^{13}C	6	7	13
		^{14}C	6	8	14
(c)	magnesium	^{24}Mg	12	12	24
		^{25}Mg	12	13	25
		^{26}Mg	12	14	26
(d)	zirconium	^{90}Zr	40	50	90
		^{91}Zr	40	51	91
		^{92}Zr	40	52	92

See pages 6–7, 46–50, and R2.2–R2.3 in *PoC*, as well as Examples 2-2 and 21-1.

2. An inversion of the preceding exercise. The chemical identity of an isotope $^{A}_{Z}X$ is determined by its atomic number Z, the number of protons. The mass number A is equal to the sum of protons and neutrons:

$$A = Z + N$$

Each of the specified isotopes has the same mass number but a different atomic number:

	Z	N	A	$^{A}_{Z}X$	Element
(a)	86	123	209	$^{209}_{86}Rn$	radon
(b)	87	122	209	$^{209}_{87}Fr$	francium
(c)	88	121	209	$^{209}_{88}Ra$	radium
(d)	89	120	209	$^{209}_{89}Ac$	actinium

See pages 6–7, 46–50, and R2.2–R2.3 in *PoC*, as well as Examples 2-2 and 21-1.

3. First, substitute the masses

$$m_{35} = 34.968852 \text{ u} \qquad m_{37} = 36.965903 \text{ u} \qquad m_{Cl} = 35.453 \text{ u}$$

into the relationship connecting the average mass of chlorine (m_{Cl}) with the mole fractions of ^{35}Cl and ^{37}Cl (X_{35} and X_{37}, respectively):

$$X_{35}\,m_{35} + X_{37}\,m_{37} = m_{\text{Cl}}$$

$$X_{35}(34.968852 \text{ u}) + X_{37}(36.965903 \text{ u}) = 35.453 \text{ u}$$

Next, use the conservation relationship

$$X_{35} + X_{37} = 1$$

to express X_{35} in terms of X_{37}:

$$X_{35} = 1 - X_{37}$$

Finally, solve the resulting equation for X_{37}:

$$(1 - X_{37})(34.968852) + X_{37}(36.965903) = 35.453$$

$$X_{37} = 0.24243$$

$$X_{35} = 1 - X_{37} = 0.75757$$

Rounded off to the third decimal place, the values are $X_{35} = 0.758$ and $X_{37} = 0.242$. See also page 68 in *PoC*.

4. Similar to the preceding exercise. We are given the quantities

$m_{24} = 23.985042$ u $\quad m_{25} = 24.985837$ u $\quad m_{26} = 25.982593$ u

$X_{24} = 0.7899 \quad X_{25} = ? \quad X_{26} = ?$

$$m_{\text{Mg}} = 24.305 \text{ u}$$

where m_A and X_A denote the mass and mole fraction, respectively, of the isotope ^{A}Mg. The symbol m_{Mg} represents the average mass of Mg at natural abundance.

The mole fractions are related as

$$X_{24} + X_{25} + X_{26} = 1$$

so that

$$0.7899 + X_{25} + X_{26} = 1$$

$$X_{25} = 0.2101 - X_{26}$$

We then substitute X_{24} (given as 0.7899), X_{25} (equal to $0.2101 - X_{26}$), and the known values of m_A into the equation for the average mass:

$$m_{\text{Mg}} = X_{24}m_{24} + X_{25}m_{25} + X_{26}m_{26}$$

$$24.305 = 0.7899(23.985042) + (0.2101 - X_{26})(24.985837) + X_{26}(25.982593)$$

$$24.305 = 24.195309 + 0.996756X_{26}$$

$$X_{26} = 0.110 \qquad X_{25} = 0.2101 - X_{26} = 0.100 \qquad X_{24} = 0.7899 \text{ (given)}$$

5. The mass of naturally occurring carbon is a weighted average that reflects the mole fractions of the stable isotopes ^{12}C and ^{13}C:

$$m_C = X_{12}\,m_{12} + X_{13}\,m_{13}$$

Given values for the mole fractions and isotopic masses,

$$X_{12} = 0.9890 \qquad X_{13} = 1 - X_{12} = 0.0110$$

$$m_{12} = 12.000000 \text{ u} \qquad m_{13} = 13.003355 \text{ u}$$

we recover the average mass found in the periodic table:

$$m_C = (0.9890)(12.000000 \text{ u}) + (0.0110)(13.003355) = 12.011$$

Our answer, rounded off to the second decimal place, is 12.01. See also page 68 in *PoC.*

Exercises 6 through 13, dealing with nuclear strength and stability, derive mostly from Section 21-4.

6. Mass defect and binding energy are covered on pages 767–769 of *PoC* and illustrated in Example 21-2.

(a) The missing mass represents the *mass defect*—the quantity of mass converted into energy during the binding of the nucleons:

$$\Delta m = \text{mass of bound nucleons} - \text{mass of separated nucleons}$$

For the mass of the bound nucleons, we subtract the mass of the single extranuclear electron from the total mass of the $^{2}_{1}H$ atom (nucleus plus electron). Note that we shall routinely retain extra digits to avoid round-off errors and loss of accuracy:

$$\text{Mass of } {}^{2}_{1}\text{H nucleus} = \text{mass of } {}^{2}_{1}\text{H atom} - \text{mass of electron}$$

$$= 2.0140 \text{ u} - 0.0005485799 \text{ u}$$

$$= 2.013451 \text{ u}$$

The mass defect is then

$$\Delta m = \text{mass of bound nucleons} - \text{mass of separated nucleons}$$

$$= 2.013451 \text{ u} - 2.01594137 \text{ u}$$

$$= -0.002490 \text{ u}$$

(b) Use Einstein's mass–energy relationship

$$\Delta E = (\Delta m)c^2$$

to compute the total binding energy. Recall that $1 \text{ J} = 1 \text{ kg m}^{-2} \text{ s}^{-2}$:

$$\Delta E = \left(-0.002490 \text{ u} \times \frac{1.660540 \times 10^{-27} \text{ kg}}{\text{u}}\right)\left(2.99792458 \times 10^{8} \text{ m s}^{-1}\right)^2$$

$$= -3.716 \times 10^{-13} \text{ J} \quad \text{(per nucleus)} \times \left(\frac{1 \text{ kJ}}{1000 \text{ J}} \times \frac{6.0221367 \times 10^{23}}{\text{mol}}\right)$$

$$= -2.238 \times 10^{8} \text{ kJ mol}^{-1}$$

(c) The binding energy per nucleon, stated as a positive number, is therefore 1.858×10^{-13} J:

$$E_A = \frac{|\Delta E|}{A} = \frac{3.716 \times 10^{-13} \text{ J}}{2 \text{ nucleons}} = \frac{1.858 \times 10^{-13} \text{ J}}{\text{nucleon}}$$

7. Similar to the preceding exercise.

(a) Since the ^{1}H nucleus contains just a single proton and no neutrons, its binding energy is zero according to our definition: the difference in energy (or mass) between the bound nucleus and its nucleons taken separately.

(b) See Exercise 6 for the calculation of Δm and ΔE for ^{2}H:

$$\Delta m = -0.002490 \text{ u}$$

$$\Delta E = (\Delta m)c^2 = -3.716 \times 10^{-13} \text{ J} \quad \text{(per nucleus)}$$

$$E_A = 1.858 \times 10^{-13} \text{ J} \quad \text{(per nucleon)}$$

(c) Tritium ($^3_1$H). The mass of the one proton and two neutrons taken separately is 3.024606 u:

$$m_p + 2m_n = 1.00727647 \text{ u} + 2 \times 1.00866490 \text{ u} = 3.024606 \text{ u}$$

The mass of a bare tritium nucleus (no electron outside) is 3.015501 u:

$$\text{Atomic mass} - \text{mass of electron} = \text{nuclear mass}$$
$$3.01605 \text{ u} - 0.0005485799 \text{ u} = 3.015501 \text{ u}$$

From the difference of these masses, we then have the mass defect and the binding energy.

Mass defect:

$$\Delta m = \text{mass of bound nucleons} - \text{mass of separated nucleons}$$
$$= 3.015501 \text{ u} - 3.024606 \text{ u}$$
$$= -0.009105 \text{ u}$$

Binding energy:

$$\Delta E = (\Delta m)c^2$$
$$= \left(-0.009105 \text{ u} \times \frac{1.660540 \times 10^{-27} \text{ kg}}{\text{u}}\right)\left(2.99792458 \times 10^8 \text{ m s}^{-1}\right)^2$$
$$= -1.359 \times 10^{-12} \text{ J} \quad \text{(per nucleus)}$$

$$E_A = \frac{|\Delta E|}{A} = \frac{1.3588 \times 10^{-12} \text{ J}}{3 \text{ nucleons}} = \frac{4.529 \times 10^{-13} \text{ J}}{\text{nucleon}}$$

8. Use the same method as in the preceding two exercises.

(a) The isotope ^{12}C contains six protons, six neutrons, and six electrons:

$$\text{Mass of bound nucleus} = \text{atomic mass} - 6m_e$$

$$= 12.000000\ \text{u} - 6(0.0005485799)\ \text{u}$$

$$= 11.9967085\ \text{u}$$

$$\text{Mass of separated nucleons} = 6m_p + 6m_n$$

$$= 6(1.00727647 + 1.00866490)\ \text{u}$$

$$= 12.0956482\ \text{u}$$

Mass defect:

$$\Delta m = \text{mass of bound nucleus} - \text{mass of separated nucleons}$$

$$= 11.9967085\ \text{u} - 12.0956482\ \text{u}$$

$$= -0.098940\ \text{u}$$

Binding energy:

$$\Delta E = (\Delta m)c^2$$

$$= \left(-0.098940\ \text{u} \times \frac{1.660540 \times 10^{-27}\ \text{kg}}{\text{u}}\right)\left(2.99792458 \times 10^{8}\ \text{m s}^{-1}\right)^2$$

$$= -1.477 \times 10^{-11}\ \text{J} \quad \text{(per nucleus)}$$

$$E_A = \frac{|\Delta E|}{A} = \frac{1.476599 \times 10^{-11}\ \text{J}}{12\ \text{nucleons}} = \frac{1.230 \times 10^{-12}\ \text{J}}{\text{nucleon}}$$

(b) Use the same method as above. The isotope ^{13}C contains six protons, seven neutrons, and six electrons:

$$\Delta m = -0.104250 \text{ u}$$

$$\Delta E = -1.556 \times 10^{-11} \text{ J} \quad \text{(per nucleus)}$$

$$E_A = 1.197 \times 10^{-12} \text{ J} \quad \text{(per nucleon)}$$

(c) Similar. Carbon-14 contains eight neutrons:

$$\Delta m = -0.113029 \text{ u}$$

$$\Delta E = -1.687 \times 10^{-11} \text{ J} \quad \text{(per nucleus)}$$

$$E_A = 1.205 \times 10^{-12} \text{ J} \quad \text{(per nucleon)}$$

9. Use the same method as in the preceding three exercises.

(a) The isotope ^{235}U contains 92 protons, 143 neutrons, and 92 electrons:

$$\Delta m = -1.915061 \text{ u}$$

$$\Delta E = -2.858 \times 10^{-10} \text{ J} \quad \text{(per nucleus)}$$

$$E_A = 1.216 \times 10^{-12} \text{ J} \quad \text{(per nucleon)}$$

(b) The isotope ^{238}U contains 92 protons, 146 neutrons, and 92 electrons:

$$\Delta m = -1.934196 \text{ u}$$

$$\Delta E = -2.887 \times 10^{-10} \text{ J} \quad \text{(per nucleus)}$$

$$E_A = 1.213 \times 10^{-12} \text{ J} \quad \text{(per nucleon)}$$

(c) The isotope ^{239}U contains 92 protons, 147 neutrons, and 92 electrons:

$$\Delta m = -1.939356 \text{ u}$$

$$\Delta E = -2.894 \times 10^{-10} \text{ J} \quad \text{(per nucleus)}$$

$$E_A = 1.211 \times 10^{-12} \text{ J} \quad \text{(per nucleon)}$$

(d) The isotope ^{240}U contains 92 protons, 148 neutrons, and 92 electrons:

$$\Delta m = -1.945723 \text{ u}$$

$$\Delta E = -2.904 \times 10^{-10} \text{ J} \quad \text{(per nucleus)}$$

$$E_A = 1.210 \times 10^{-12} \text{ J} \quad \text{(per nucleon)}$$

10. The strength of binding has no influence on the rate of decay. Binding energy, like the more familiar Gibbs free energy in a chemical reaction, is a thermodynamic quantity independent of time. The rate of decay involves, instead, questions of kinetics and mechanism.

11. No, there is no connection between half-life and binding energy. Half-life is a kinetic issue, not a measure of thermodynamic stability.

12. See pages 772–773 in *PoC* and also the comments accompanying Exercise 13 below. Magic numbers (2, 8, 20, 28, 50, 82, 126) are indicated by an asterisk in the following table:

NUCLEUS	A	Z	N
$^{16}_{8}O$	16	8*	8*
$^{17}_{8}O$	17	8*	9
$^{16}_{9}F$	16	9	7
$^{19}_{9}F$	19	9	10
$^{40}_{20}Ca$	40	20*	20*
$^{40}_{21}Sc$	40	21	19
$^{208}_{82}Pb$	208	82*	126*

13. See pages 772–773 and R21.2–R21.3 in *PoC*, together with Example 21-1 and Exercises 1 and 2. For each nucleus $^{A}_{Z}X$, the mass number A is equal to the number of protons (Z) plus the number of neutrons (N):

$$A = Z + N$$

A magic number (2, 8, 20, 28, 50, 82, 126 neutrons or protons) corresponds to a filled nuclear shell. The more stable nucleus in each pair has a magic number of protons or neutrons or both, indicated by asterisks in the table that follows:

	NUCLEUS	A	Z	N	COMMENT
(a)	$^{3}_{2}He$	3	2^*	1	less stable
	$^{4}_{2}He$	4	2^*	2^*	more stable
(b)	$^{206}_{82}Pb$	206	82^*	124	less stable
	$^{208}_{82}Pb$	208	82^*	126^*	more stable
(c)	$^{208}_{83}Bi$	208	83	125	less stable
	$^{209}_{83}Bi$	209	83	126^*	more stable

Nuclear transformations are the subject now of a large block of problems, running from Exercise 14 through Exercise 34. See page 754, Section 21-5, and Examples 21-3 through 21-5 in PoC.

14. Emission of an α particle, $^{4}_{2}He$, reduces the atomic number by 2 and the mass number by 4 (see Figure 21-10a on page 777 of *PoC*). Each mother nucleus stands two positions to the right of her daughter in the periodic table.

	MOTHER	DAUGHTER
(a)	polonium-193	lead-189
(b)	actinium-222	francium-218
(c)	radium-222	radon-218
(d)	lawrencium-253	mendelevium-249

Full decay equations and symbols are given in the next exercise.

15. Equations for α decay are illustrated on page 780 and in Example 21-3 of *PoC*:

$$^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}X' + {}^{4}_{2}He$$

The atomic number decreases by 2 and the mass number decreases by 4.

(a) $^{193}_{84}Po \rightarrow {}^{189}_{82}Pb + {}^{4}_{2}He$

(b) $^{222}_{89}Ac \rightarrow {}^{218}_{87}Fr + {}^{4}_{2}He$

(c) $^{222}_{88}Ra \rightarrow ^{218}_{86}Rn + ^{4}_{2}He$

(d) $^{253}_{103}Lr \rightarrow ^{249}_{101}Md + ^{4}_{2}He$

16. The mother's atomic number is greater than the daughter's by 2. The mother's mass number is greater by 4.

	MOTHER	DAUGHTER
(a)	uranium-222	thorium-218
(b)	protactinium-222	actinium-218
(c)	thorium-217	radium-213
(d)	californium-240	curium-236

Each mother stands two positions to the right of her daughter in the periodic table. See the next exercise for the full decay equations.

17. Use the same method as in Exercise 15.

(a) $^{222}_{92}U \rightarrow ^{218}_{90}Th + ^{4}_{2}He$

(b) $^{222}_{91}Pa \rightarrow ^{218}_{89}Ac + ^{4}_{2}He$

(c) $^{217}_{90}Th \rightarrow ^{213}_{88}Ra + ^{4}_{2}He$

(d) $^{240}_{98}Cf \rightarrow ^{236}_{96}Cm + ^{4}_{2}He$

18. Emission of a β^- particle, $^{0}_{-1}e$, increases the atomic number by 1 while leaving the mass number unchanged (see Figure 21b on page 777 of *PoC*). Each mother nucleus stands one position to the left of her daughter in the periodic table.

	MOTHER	DAUGHTER
(a)	barium-142	lanthanum-142
(b)	cesium-137	barium-137
(c)	carbon-14	nitrogen-14
(d)	boron-12	carbon-12

Full equations are given in the next exercise.

19. A neutron emits a β^- particle, ${}^{0}_{-1}e$, and is converted into a proton:

$$ {}^{1}_{0}n \rightarrow {}^{1}_{1}p + {}^{0}_{-1}e + \tilde{\nu} $$

An uncharged and presumably massless *antineutrino*, $\tilde{\nu}$, is released as well.

For β^- decay within a nucleus ${}^{A}_{Z}X$, we represent the general process by the following equation:

$$ {}^{A}_{Z}X \rightarrow {}_{Z+1}^{A}X' + {}^{0}_{-1}e + \tilde{\nu} $$

See pages 776–779 and Example 21-3 in *PoC.*

(a) $ {}^{142}_{56}Ba \rightarrow {}^{142}_{57}La + {}^{0}_{-1}e + \tilde{\nu} $

(b) $ {}^{137}_{55}Cs \rightarrow {}^{137}_{56}Ba + {}^{0}_{-1}e + \tilde{\nu} $

(c) $ {}^{14}_{6}C \rightarrow {}^{14}_{7}N + {}^{0}_{-1}e + \tilde{\nu} $

(d) $ {}^{12}_{5}B \rightarrow {}^{12}_{6}C + {}^{0}_{-1}e + \tilde{\nu} $

20. The mother's atomic number is less than the daughter's by 1. The mass number is the same for both mother and daughter.

	MOTHER	DAUGHTER
(a)	holmium-168	erbium-168
(b)	berkelium-251	californium-251
(c)	nitrogen-16	oxygen-16
(d)	sulfur-35	chlorine-35

Each mother nucleus stands one position to the left of her daughter in the periodic table. See pages 776–779 and Example 21-3 in *PoC.*

Full decay equations and symbols are given in the next exercise.

21. Use the same method as in Exercise 19.

(a) $ {}^{168}_{67}Ho \rightarrow {}^{168}_{68}Er + {}^{0}_{-1}e + \tilde{\nu} $

(b) $ {}^{251}_{97}Bk \rightarrow {}^{251}_{98}Cf + {}^{0}_{-1}e + \tilde{\nu} $

(c) $^{16}_{7}N \rightarrow {}^{16}_{8}O + {}^{0}_{-1}e + \tilde{\nu}$

(d) $^{35}_{16}S \rightarrow {}^{35}_{17}Cl + {}^{0}_{-1}e + \tilde{\nu}$

22. Emission of an α particle reduces Z by 2 and A by 4. Emission of a β^- particle increases Z by 1 and leaves A unchanged. The full series is specified below:

$$^{232}_{90}Th \xrightarrow{\alpha} {}^{228}_{88}Ra \xrightarrow{\beta^-} {}^{228}_{89}Ac \xrightarrow{\beta^-} {}^{228}_{90}Th \xrightarrow{\alpha} {}^{224}_{88}Ra \xrightarrow{\alpha}$$

$$^{220}_{86}Rn \xrightarrow{\alpha} {}^{216}_{84}Po \xrightarrow{\alpha} {}^{212}_{82}Pb \xrightarrow{\beta^-} {}^{212}_{83}Bi \xrightarrow{\beta^-} {}^{212}_{84}Po \xrightarrow{\alpha} {}^{208}_{82}Pb$$

23. Uranium-235 loses 28 units of mass number during its transformation from $^{235}_{92}U$ to $^{207}_{82}Pb$. To do so, it must emit seven α particles—a process that would lower the atomic number from 92 to 78. Release of four β^- particles then raises the atomic number to 82:

$$^{235}_{92}U \rightarrow {}^{207}_{82}Pb + 7\,{}^{4}_{2}He + 4\,{}^{0}_{-1}e + 4\tilde{\nu}$$

24. Look at the periodic table—the mother nucleus stands one position to the right of her daughter in all processes where the atomic number (Z) decreases by 1:

	Mother	Daughter	Process	ΔZ	ΔA
(a)	arsenic-73	germanium-73	electron capture	−1	0
(b)	fluorine-15	oxygen-14	proton emission	−1	−1
(c)	helium-5	helium-4	neutron emission	0	−1
(d)	aluminum-24	magnesium-24	positron emission	−1	0
(e)	vanadium-49	titanium-49	electron capture	−1	0

The various transformations are described on pages 776–780 and in Example 21-4 of *PoC*. See the next exercise for the corresponding equations.

25. Equations for the processes in Exercise 24.

(a) $^{73}_{33}As + {}^{0}_{-1}e \rightarrow {}^{73}_{32}Ge + \nu$

(b) $^{15}_{9}F \rightarrow {}^{14}_{8}O + {}^{1}_{1}p$

(c) $^{5}_{2}He \rightarrow {}^{4}_{2}He + {}^{1}_{0}n$

(d) $^{24}_{13}\text{Al} \rightarrow\ ^{24}_{12}\text{Mg} +\ ^{0}_{1}\text{e} + \nu$

(e) $^{49}_{23}\text{V} +\ ^{0}_{-1}\text{e} \rightarrow\ ^{49}_{22}\text{Ti} + \nu$

26. Similar to Exercise 24. The mother nucleus stands *n* positions to the right of her daughter in all processes where the atomic number (Z) decreases by *n*:

	MOTHER	DAUGHTER	PROCESS	ΔZ	ΔA
(a)	titanium-45	scandium-45	electron capture	−1	0
(b)	titanium-45	scandium-45	positron emission	−1	0
(c)	neon-16	oxygen-14	proton emission (×2)	−2	−2
(d)	helium-7	helium-6	neutron emission	0	−1
(e)	lanthanum-135	barium-135	electron capture	−1	0

The various transformations are described on pages 776–780 and in Example 21-4 of *PoC*. See the next exercise for the corresponding equations.

27. Equations for the processes in Exercise 26.

(a) $^{45}_{22}\text{Ti} +\ ^{0}_{-1}\text{e} \rightarrow\ ^{45}_{21}\text{Sc} + \nu$

(b) $^{45}_{22}\text{Ti} \rightarrow\ ^{45}_{21}\text{Sc} +\ ^{0}_{1}\text{e} + \nu$

(c) $^{16}_{10}\text{Ne} \rightarrow\ ^{14}_{8}\text{O} + 2\,^{1}_{1}\text{p}$

(d) $^{7}_{2}\text{He} \rightarrow\ ^{6}_{2}\text{He} +\ ^{1}_{0}\text{n}$

(e) $^{135}_{57}\text{La} +\ ^{0}_{-1}\text{e} \rightarrow\ ^{135}_{56}\text{Ba} + \nu$

28. Similar to Exercise 24. The mother nucleus stands one position to the right of her daughter in all processes where the atomic number (Z) decreases by 1:

	MOTHER	DAUGHTER	PROCESS	ΔZ	ΔA
(a)	promethium-143	neodymium-143	electron capture	−1	0
(b)	iodine-109	tellurium-108	proton emission	−1	−1
(c)	tin-103	indium-103	positron emission	−1	0
(d)	helium-9	helium-8	neutron emission	0	−1
(e)	calcium-35	potassium-35	positron emission	−1	0

The various transformations are described on pages 776–780 and in Example 21-4 of *PoC*. See the next exercise for the corresponding equations.

29. Equations for the processes in Exercise 28.

(a) $^{143}_{61}\text{Pm} + ^{0}_{-1}\text{e} \rightarrow ^{143}_{60}\text{Nd} + \nu$

(b) $^{109}_{53}\text{I} \rightarrow ^{108}_{52}\text{Te} + ^{1}_{1}\text{p}$

(c) $^{103}_{50}\text{Sn} \rightarrow ^{103}_{49}\text{In} + ^{0}_{1}\text{e} + \nu$

(d) $^{9}_{2}\text{He} \rightarrow ^{8}_{2}\text{He} + ^{1}_{0}\text{n}$

(e) $^{35}_{20}\text{Ca} \rightarrow ^{35}_{19}\text{K} + ^{0}_{1}\text{e} + \nu$

30. Balance the number of protons and the total mass number on each side of the equation.

(a) Nitrogen-14 absorbs an α particle and then produces oxygen-17 by proton emission:

$$^{14}_{7}\text{N} + ^{4}_{2}\text{He} \rightarrow ^{17}_{8}\text{O} + ^{1}_{1}\text{H}$$

The symbols $^{1}_{1}\text{H}$ and $^{1}_{1}\text{p}$ are equivalent.

(b) Cobalt-59 absorbs a neutron and becomes cobalt-60:

$$^{59}_{27}\text{Co} + ^{1}_{0}\text{n} \rightarrow ^{60}_{27}\text{Co}$$

(c) Carbon-14 undergoes β^- decay, producing nitrogen-14:

$$^{14}_{6}\text{C} \rightarrow ^{14}_{7}\text{N} + ^{0}_{-1}\text{e} + \tilde{\nu}$$

(d) Uranium-235 undergoes fission after absorbing a neutron, splitting into tellurium-137 and zirconium-97. Two neutrons are released:

$$^{235}_{92}\text{U} + ^{1}_{0}\text{n} \rightarrow ^{137}_{52}\text{Te} + ^{97}_{40}\text{Zr} + 2\,^{1}_{0}\text{n}$$

(e) Plutonium-239 absorbs an α particle and then produces curium-242 by neutron emission:

$$^{239}_{94}\text{Pu} + \alpha \rightarrow ^{242}_{96}\text{Cm} + ^{1}_{0}\text{n}$$

The symbols $^{4}_{2}\text{He}$ and α are equivalent.

31. The method is the same as in the preceding exercise. Balance the number of protons and the total mass number on each side of the equation.

(a) Oxygen-16 absorbs a proton and then produces nitrogen-13 by α decay, emitting a helium-4 nucleus:

$$^{16}_{8}\text{O} + ^{1}_{1}\text{p} \rightarrow ^{13}_{7}\text{N} + ^{4}_{2}\text{He}$$

(b) Aluminum-27 absorbs a neutron and then produces sodium-24 by α decay:

$$^{27}_{13}\text{Al} + ^{1}_{0}\text{n} \rightarrow ^{24}_{11}\text{Na} + ^{4}_{2}\text{He}$$

(c) Iron-58 absorbs a neutron and becomes iron-59:

$$^{58}_{26}\text{Fe} + ^{1}_{0}\text{n} \rightarrow ^{59}_{26}\text{Fe}$$

(d) Helium-3 and hydrogen-1 undergo fusion to produce helium-4. A positron and a neutrino (total lepton number = 0) are released as by-products of the fusion process:

$$^{3}_{2}\text{He} + ^{1}_{1}\text{H} \rightarrow ^{4}_{2}\text{He} + ^{0}_{1}\text{e} + \nu$$

(e) Uranium-235 undergoes fission after absorbing a neutron, splitting into barium-142 and krypton-91. Three neutrons are released:

$$^{235}_{92}\text{U} + ^{1}_{0}\text{n} \rightarrow ^{142}_{56}\text{Ba} + ^{91}_{36}\text{Kr} + 3\,^{1}_{0}\text{n}$$

32. Conservation of particle number is discussed on pages 777–779, R21.9–R21.11, and R21.20 of *PoC*.

(a) The process

$$^{1}_{0}\text{n} + ^{1}_{0}\text{n} \rightarrow ^{1}_{1}\text{p} + ^{1}_{-1}\text{p} \qquad \text{(forbidden)}$$

would violate conservation of baryon number:

Left-hand side:	Baryon number = 2	(contributed by 2 neutrons)
Right-hand side:	Baryon number = 0	(1 proton and 1 antiproton)

(b) The process

$$^{238}_{92}\text{U} \rightarrow\ ^{233}_{90}\text{Th} +\ ^{4}_{2}\text{He} +\ ^{1}_{1}\text{p} \qquad \text{(forbidden)}$$

would violate conservation of charge:

Left-hand side:	Charge = 92
Right-hand side:	Charge = 90 + 2 + 1 = 93

(c) The process

$$^{59}_{26}\text{Fe} \rightarrow\ ^{59}_{27}\text{Co} +\ ^{0}_{-1}\text{e} + \nu \qquad \text{(forbidden)}$$

would violate conservation of lepton number:

Left-hand side:	Lepton number = 0	(no leptons)
Right-hand side:	Lepton number = 2	(electron plus neutrino)

(d) The process

$$^{0}_{-1}\text{e} +\ ^{0}_{1}\text{e} \rightarrow \nu \qquad \text{(forbidden)}$$

would violate conservation of lepton number:

Left-hand side:	Lepton number = 0	(electron and positron)
Right-hand side:	Lepton number = 1	(neutrino)

Photons—not a neutrino—are produced by the annihilation of an electron and positron.

33. The criterion for spontaneity is a net decrease in mass. We compute Δm for each reaction, taking $m_e = 0.0005485799$ u as the mass of an electron or positron. A mass of zero for neutrinos and antineutrinos is assumed throughout.

(a) Positron emission is not spontaneous for plutonium-234. The total mass increases:

$$^{234}_{94}\text{Pu} \rightarrow\ ^{234}_{93}\text{Np} +\ ^{0}_{1}\text{e} + \nu$$

$$\Delta m = m(^{234}\text{Np}) + 2m_e - m(^{234}\text{Pu})$$

$$= 234.042888 \text{ u} + 2(0.0005485799 \text{ u}) - 234.043299 \text{ u}$$

$$= +0.000686 \text{ u} \quad \text{(nonspontaneous)}$$

The masses are *atomic* masses. We add an extra value of m_e on the right (in addition to the positron) and thereby account for the 94th electron originally outside the $^{234}_{94}\text{Pu}$ nucleus.

Note that the equivalent calculation of $\Delta m(\beta^+)$ in Example 21-5 is shown incorrectly in some printings of *PoC*, inadvertently omitting the second electronic mass on the right-hand side.

(b) Electron capture by plutonium-234 is allowed, even though positron emission is not. The change in mass, Δm, is negative:

$$^{234}_{94}\text{Pu} + {}^{0}_{-1}\text{e} \rightarrow {}^{234}_{93}\text{Np} + \nu$$

$$\Delta m = m(^{234}\text{Np}) - m(^{234}\text{Pu})$$

$$= 234.042888 \text{ u} - 234.043299 \text{ u}$$

$$= -0.000411 \text{ u} \quad \text{(spontaneous)}$$

Observe that the mass of the captured electron is already included in the atomic mass of ^{234}Pu.

(c) Neutron emission is not spontaneous for carbon-12:

$$^{12}_{6}\text{C} \rightarrow {}^{11}_{6}\text{C} + {}^{1}_{0}\text{n}$$

$$\Delta m = m(^{11}\text{C}) + m_{\text{n}} - m(^{12}\text{C})$$

$$= 11.01143 \text{ u} + 1.0086649 \text{ u} - 12.000000 \text{ u}$$

$$= +0.02009 \text{ u} \quad \text{(nonspontaneous)}$$

(d) Electron emission is spontaneous for beryllium-10:

$$^{10}_{4}\text{Be} \rightarrow {}^{10}_{5}\text{B} + {}^{0}_{-1}\text{e} + \tilde{\nu}$$

$$\Delta m = m(^{10}\text{B}) - m(^{10}\text{Be})$$

$$= 10.012937 \text{ u} - 10.013534 \text{ u}$$

$$= -0.000597 \text{ u} \quad \text{(spontaneous)}$$

The mass of the emitted electron is automatically included in the atomic mass of ^{10}B.

34. Compute Δm using the same techniques as in the preceding exercise.

(a) Silver-98 undergoes spontaneous positron emission:

$$^{98}_{47}\text{Ag} \rightarrow {}^{98}_{46}\text{Pd} + {}^{0}_{1}\text{e} + \nu$$

$$\Delta m = m(^{98}\text{Pd}) + 2m_{\text{e}} - m(^{98}\text{Ag})$$

$$= 97.912722 \text{ u} + 2(0.0005485799 \text{ u}) - 97.921560 \text{ u}$$

$$= -0.007741 \text{ u} \quad \text{(spontaneous)}$$

(b) Electron capture is possible as well for silver-98:

$$^{98}_{47}\text{Ag} + {}^{0}_{-1}\text{e} \rightarrow {}^{98}_{46}\text{Pd} + \nu$$

$$\Delta m = m(^{98}\text{Pd}) - m(^{98}\text{Ag})$$

$$= 97.912722\ \text{u} - 97.921560\ \text{u}$$

$$= -0.008838\ \text{u} \quad \text{(spontaneous)}$$

(c) Silver-98 does not undergo spontaneous electron emission:

$$^{98}_{47}\text{Ag} \rightarrow {}^{98}_{48}\text{Cd} + {}^{0}_{-1}\text{e} + \tilde{\nu}$$

$$\Delta m = m(^{98}\text{Cd}) - m(^{98}\text{Ag})$$

$$= 97.92711\ \text{u} - 97.921560\ \text{u}$$

$$= +0.00555\ \text{u} \quad \text{(nonspontaneous)}$$

(d) Alpha emission is not allowed for silver-98. The total mass increases:

$$^{98}_{47}\text{Ag} \rightarrow {}^{94}_{45}\text{Rh} + {}^{4}_{2}\text{He}$$

$$\Delta m = m(^{94}\text{Rh}) + m(^{4}\text{He}) - m(^{98}\text{Ag})$$

$$= 93.921670\ \text{u} + 4.002603\ \text{u} - 97.921560\ \text{u}$$

$$= +0.002713\ \text{u} \quad \text{(nonspontaneous)}$$

First-order nuclear decay is discussed on pages 774–776 and R21.3–R21.4 of PoC, *with accompanying sample calculations in Examples 21-6 and 21-7. First-order chemical processes, described by exactly the same mathematics, are treated in Chapter 18 (pages 653–656, Examples 18-4 and 18-5, and Exercises 21 through 27).*

35. All first-order decay processes are characterized by a rate constant k and an associated half-life $t_{1/2}$:

$$k = \frac{\ln 2}{t_{1/2}}$$

Knowing either k or $t_{1/2}$, we can always evaluate the decay profile:

$$\frac{N_t}{N_0} = \exp(-kt) = \exp\left[-\left(\frac{\ln 2}{t_{1/2}}\right)t\right]$$

N_0, used here to mean the initial population, is not to be confused with Avogadro's number (which is represented by the same symbol in other contexts). See also Examples 21-6 and 21-7.

(a) Substitute, for instance, $t = 2500$ y and $t_{1/2} = 5730$ y into the equation given above:

$$\frac{N_{2500\text{ y}}}{N_0} = \exp(-kt) = \exp\left[-\left(\frac{\ln 2}{t_{1/2}}\right)t\right]$$

$$= \exp\left[-\left(\frac{0.6931}{5730\text{ y}}\right) \times 2500\text{ y}\right]$$

$$= 0.739$$

Results are summarized in the table below:

TIME (y)	N_t/N_0
2500	0.739
5000	0.546
10,000	0.298
20,000	0.089

(b) Write the equation

$$\frac{N_t}{N_0} = \exp(-kt) = \exp\left[-\left(\frac{\ln 2}{t_{1/2}}\right)t\right]$$

and take the natural logarithm of both sides. Then solve for t when $N_t/N_0 = 0.100$:

$$\ln\left(\frac{N_t}{N_0}\right) = -\left(\frac{\ln 2}{t_{1/2}}\right)t$$

$$t = -\left(\frac{t_{1/2}}{\ln 2}\right)\ln\left(\frac{N_t}{N_0}\right) = -\left(\frac{5730\text{ y}}{0.6931}\right)\ln 0.100 = 1.90 \times 10^4\text{ y}$$

36. Use the same method as in the preceding exercise.

(a) Given the half-life and initial population,

$$t_{1/2} = 8.8 \text{ h} \qquad N_0 = 1{,}000{,}000$$

we solve for the population N_t at time t:

$$N_t = N_0 \exp\left[-\left(\frac{\ln 2}{t_{1/2}}\right)t\right] = (1{,}000{,}000)\exp\left[-\left(\frac{0.6931}{8.8 \text{ h}}\right)t\right]$$

Results are tabulated below:

TIME (h)	N_0
2.0	8.5×10^5
4.0	7.3×10^5
8.0	5.3×10^5
16.0	2.8×10^5
32.0	8.0×10^4

(b) Solve for t when $N_t = 976{,}432$:

$$t = -\left(\frac{t_{1/2}}{\ln 2}\right)\ln\left(\frac{N_t}{N_0}\right) = -\left(\frac{8.8 \text{ h}}{0.6931}\right)\ln\left(\frac{976{,}432}{1{,}000{,}000}\right) = 0.30 \text{ h}$$

37. Expressed logarithmically, the activity profile

$$A_t = A_0 \exp(-kt)$$

becomes

$$\ln A_t = -kt + \ln A_0$$

A plot of $\ln A_t$ versus t will produce a straight line with slope equal to $-k$ and intercept equal to $\ln A_0$. Once the slope is obtained from the plot, the half-life follows directly:

$$k = -\text{slope} = \frac{\ln 2}{t_{1/2}}$$

Alternatively, a plot of $\ln(A_t/A_0)$ will produce a straight line with the same slope ($-k$) but with intercept equal to 0. See page R21.4 in *PoC*.

38. Activity is defined on page R21.4 and demonstrated in Example 21-7. See the QUESTION/ANSWER dialogue on pages R21.17–21.18 and also the solution to Exercise 37, immediately above.

(a) We plot $\ln(A_t/A_0)$ versus t (Figure 21.1) and determine the rate constant from the slope of the resulting straight line:

$$\text{Slope} = -0.005778 \text{ s}^{-1} = -k$$

$$k = 0.00578 \text{ s}^{-1} \quad \text{(3 sig fig)}$$

The half-life is inversely proportional to the rate constant:

$$t_{1/2} = \frac{\ln 2}{k} = \frac{0.6931}{0.005778 \text{ s}^{-1}} = 120. \text{ s}$$

A similar operation is performed in Exercise 22 of Chapter 18.

(b) From the activity at time $t = 0$,

$$A_0 = kN_0 = 1000 \text{ Bq} = 1000 \text{ s}^{-1}$$

we calculate first the initial population N_0 and then the corresponding mass in grams. Remember that N_0 is not used to represent Avogadro's number in this context.

Initial population:

$$N_0 = \frac{A_0}{k} = \frac{1000 \text{ atoms } ^{232}\text{Ac s}^{-1}}{0.005778 \text{ s}^{-1}} = 1.73 \times 10^5 \text{ atoms } ^{232}\text{Ac}$$

Initial mass:

$$\left(\frac{1000 \text{ atoms } ^{232}\text{Ac s}^{-1}}{0.005778 \text{ s}^{-1}}\right) \times \frac{1 \text{ mol } ^{232}\text{Ac}}{6.022 \times 10^{23} \text{ atoms } ^{232}\text{Ac}} \times \frac{232.04213 \text{ g } ^{232}\text{Ac}}{\text{mol } ^{232}\text{Ac}}$$

$$= 6.67 \times 10^{-17} \text{ g } ^{232}\text{Ac}$$

(c) Given the rate constant and an initial activity, we can calculate the activity at any subsequent time t (in this case, 360 s):

$$A_t = A_0 \exp(-kt) = \left(1000 \text{ s}^{-1}\right)\exp\left[-\left(0.005778 \text{ s}^{-1}\right)\left(360 \text{ s}\right)\right] = 125 \text{ s}^{-1}$$

$$= 125 \text{ Bq} \quad \text{(three half-lives)}$$

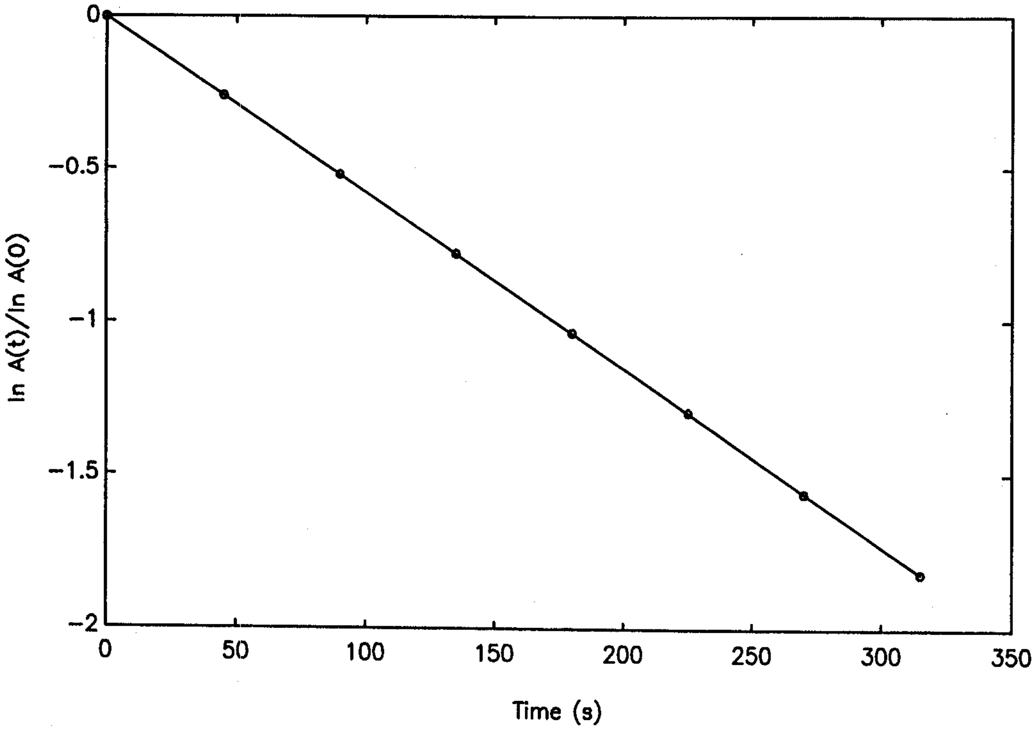

FIGURE 21.1 Semilog plot of normalized activity versus time for the process in Exercise 38. The straight line has a slope of $-0.005778\ \text{s}^{-1}$ and an intercept of 0.00060 (nearly 0), as determined by a linear regression analysis. See Exercise 38 in Chapter 18 for a brief discussion of linear regression, also called the method of least squares.

The associated mass is calculated as described in (b):

$$\left(\frac{125 \text{ atoms } ^{232}\text{Ac s}^{-1}}{0.005778 \text{ s}^{-1}}\right) \times \frac{1 \text{ mol } ^{232}\text{Ac}}{6.022 \times 10^{23} \text{ atoms } ^{232}\text{Ac}} \times \frac{232.04213 \text{ g } ^{232}\text{Ac}}{\text{mol } ^{232}\text{Ac}}$$

$$= 8.34 \times 10^{-18} \text{ g } ^{232}\text{Ac}$$

39. Another computation of activity, similar to Exercise 38.

(a) From the half-life,

$$t_{1/2} = 12.3 \text{ y} \times \frac{365.25 \text{ d}}{\text{y}} \times \frac{86{,}400 \text{ s}}{\text{d}} = 3.88 \times 10^{8} \text{ s}$$

we first calculate the rate constant:

$$k = \frac{\ln 2}{t_{1/2}}$$

The activity equation

$$A = kN$$

then enables us to determine the existing population and mass:

$$N = \frac{A}{k} = \frac{At_{1/2}}{\ln 2} = \frac{(500 \text{ atoms } ^3\text{H s}^{-1})(3.88 \times 10^8 \text{ s})}{0.6931} = 2.80 \times 10^{11} \text{ atoms } ^3\text{H}$$

$$2.80 \times 10^{11} \text{ atoms } ^3\text{H} \times \frac{1 \text{ mol } ^3\text{H}}{6.022 \times 10^{23} \text{ atoms } ^3\text{H}} \times \frac{3.01605 \text{ g } ^3\text{H}}{\text{mol } ^3\text{H}} = 1.40 \times 10^{-12} \text{ g } ^3\text{H}$$

(b) To calculate the activity at any subsequent time t, substitute t and $k = (\ln 2)/t_{1/2}$ into the first-order decay equation:

$$A_t = A_0 \exp(-kt) = A_0 \exp\left[-\left(\frac{\ln 2}{t_{1/2}}\right)t\right]$$

For example, after six years (nearly 0.5 $t_{1/2}$) the activity is slightly greater than 71% of its initial value:

$$A_t = (500 \text{ Bq})\exp\left[-\left(\frac{0.6931}{12.3 \text{ y}}\right)(6 \text{ y})\right] = 357 \text{ Bq} = 357 \text{ s}^{-1}$$

The mass in grams is obtained from N_t, the number of atoms remaining at time t:

$$N_t = \frac{A_t}{k} = \frac{(A_t)(t_{1/2})}{\ln 2} = \frac{(357 \text{ atoms } ^3\text{H s}^{-1})(3.88 \times 10^8 \text{ s})}{0.6931} = 2.00 \times 10^{11} \text{ atoms } ^3\text{H}$$

$$2.00 \times 10^{11} \text{ atoms } ^3\text{H} \times \frac{1 \text{ mol } ^3\text{H}}{6.022 \times 10^{23} \text{ atoms } ^3\text{H}} \times \frac{3.01605 \text{ g } ^3\text{H}}{\text{mol } ^3\text{H}} = 1.00 \times 10^{-12} \text{ g } ^3\text{H}$$

Results are summarized below:

TIME (y)	ACTIVITY (Bq)	MASS (g)
6	357	1.00×10^{-12}
12	254	7.12×10^{-13}
18	181	5.08×10^{-13}
24	129	3.62×10^{-13}

We state the numbers here to three significant figures, treating the values of t as exact quantities supplied for theoretical purposes. With this assumption, the three-digit accuracy of $t_{1/2}$ determines the significance of the final result.

40. Given a half-life of 8.04 days, solve the activity equation

$$\frac{A_t}{A_0} = \exp(-kt) = \exp\left[-\left(\frac{\ln 2}{t_{1/2}}\right)t\right]$$

for t when $A_t/A_0 = 0.0100$:

$$\ln\left(\frac{A_t}{A_0}\right) = -\left(\frac{\ln 2}{t_{1/2}}\right)t$$

$$t = -\left(\frac{t_{1/2}}{\ln 2}\right)\ln\left(\frac{A_t}{A_0}\right) = -\left(\frac{8.04 \text{ d}}{0.6931}\right)\ln 0.0100 = 53.4 \text{ d}$$

The calculation is similar to that undertaken in Exercise 35(b).

41. We continue to exploit the inverse relationship between the first-order rate constant and half-life:

$$k = \frac{\ln 2}{t_{1/2}}$$

(a) Emission of a β^- particle raises the atomic number by 1. Phosporus-32 decays to sulfur-32, a stable isotope:

$$^{32}_{15}\text{P} \rightarrow {}^{32}_{16}\text{S} + {}^{0}_{-1}\text{e} + \tilde{\nu}$$

(b) The mixture consists entirely of ^{32}P and ^{32}S, since there are no other routes of decay. Suppose, then, that we begin with 1,000,000 atoms of ^{32}P ($t_{1/2} = 14.28$ days), which subsequently decay over a period of 21.00 days:

$$N_t = N_0 \exp(-kt) = N_0 \exp\left[-\left(\frac{\ln 2}{t_{1/2}}\right)t\right]$$

$$= (1{,}000{,}000 \text{ atoms } ^{32}\text{P}) \exp\left[-\left(\frac{0.693147}{14.28 \text{ d}}\right)(21.00 \text{ d})\right]$$

$$= 360{,}835 \text{ atoms } ^{32}\text{P}$$

The arbitrary mixture thus contains 360,835 atoms of phosphorus-32 and 639,165 atoms of sulfur-32, from which we calculate the total mass of each isotope:

$$360{,}835 \text{ atoms } ^{32}\text{P} \times \frac{31.973907 \text{ u}}{\text{atom } ^{32}\text{P}} = 11{,}537{,}305 \text{ u } ^{32}\text{P}$$

$$639{,}165 \text{ atoms } ^{32}\text{S} \times \frac{31.972070 \text{ u}}{\text{atom } ^{32}\text{S}} = 20{,}435{,}428 \text{ u } ^{32}\text{S}$$

The mass percentages, rounded off to four significant figures, follow straightforwardly:

$$\frac{\text{mass of } ^{32}\text{P}}{\text{total mass}} \times 100\% = \frac{11{,}537{,}305 \text{ u}}{11{,}537{,}305 \text{ u} + 20{,}435{,}428 \text{ u}} \times 100\% = 36.08\% \ ^{32}\text{P} \ \text{(mass)}$$

$$\frac{\text{mass of } ^{32}\text{S}}{\text{total mass}} \times 100\% = \frac{20{,}435{,}428 \text{ u}}{11{,}537{,}305 \text{ u} + 20{,}435{,}428 \text{ u}} \times 100\% = 63.92\% \ ^{32}\text{S} \ \text{(mass)}$$

42. Since only the one isotope of lead, ^{206}Pb, appears in the rock, we assume that it has been produced entirely by the radioactive decay of ^{238}U. If not, we should expect to find a distribution of ^{204}Pb, ^{206}Pb, ^{207}Pb, and ^{208}Pb—not lead-206 alone.

The first step is to determine the mass of ^{238}U that has been converted into the present mass of ^{206}Pb:

$$0.101 \text{ g } ^{206}\text{Pb} \times \frac{238.050784 \text{ g } ^{238}\text{U}}{205.974440 \text{ g } ^{206}\text{Pb}} = 0.117 \text{ g } ^{238}\text{U}$$

The original mass of ^{238}U present at $t = 0$ is therefore 0.843 g:

$$\text{Initial mass} = 0.726 \text{ g} + 0.117 \text{ g} = 0.843 \text{ g}$$

Recognizing that N (the number of uranium atoms) is proportional to mass, we then solve the decay equation

$$\frac{N_t}{N_0} = \exp(-kt) = \exp\left[-\left(\frac{\ln 2}{t_{1/2}}\right)t\right]$$

for the elapsed time ($t = 970$ million years):

$$\ln\left(\frac{N_t}{N_0}\right) = -\left(\frac{\ln 2}{t_{1/2}}\right)t$$

$$t = -\left(\frac{t_{1/2}}{\ln 2}\right)\ln\left(\frac{N_t}{N_0}\right) = -\left(\frac{4.5\times 10^9\ \text{y}}{0.6931}\right)\ln\left(\frac{0.726\ \text{g}}{0.843\ \text{g}}\right) = 9.7\times 10^8\ \text{y}$$

See Example 21-7.

43. To clarify the numerical relationships, multiply the stated amounts by 2

^{208}Pb	0.2620 g	× 2	=	0.5240 g
^{207}Pb	0.1105 g	× 2	=	0.2210 g
^{206}Pb	0.2205 g	× 2	=	0.4410 g
^{204}Pb	0.0070 g	× 2	=	0.0140 g

and compare the resulting proportions with the natural distribution of isotopes in 1.0000 g of lead:

	Actual Mass (if doubled)		Expected Mass (if at natural abundance)
^{208}Pb	0.5240 g	=	0.5240 g
^{207}Pb	0.2210 g	=	0.2210 g
^{206}Pb	0.4410 g	>	0.2410 g
^{204}Pb	0.0140 g	=	0.0140 g
Total	1.2000 g	>	1.0000 g

Doing so, we see that lead-206 is present in excess of its natural proportions:

$$\text{Excess }^{206}\text{Pb} = \tfrac{1}{2}\times(0.4410\ \text{g} - 0.2410\ \text{g}) = 0.1000\ \text{g}$$

Thus of the 0.2205 g ^{206}Pb in the rock, fully 0.1000 g (the entire excess mass) must have come originally from 0.1156 g ^{238}U, as shown below:

$$0.1000\ \text{g}\ ^{206}\text{Pb}\times\frac{238.050784\ \text{g}\ ^{238}\text{U}}{205.974440\ \text{g}\ ^{206}\text{Pb}} = 0.1156\ \text{g}\ ^{238}\text{U converted into }^{206}\text{Pb}$$

Comparing the mass of ^{238}U currently on hand ($m_t = 0.9050$ g) with the mass present at $t = 0$,

$$m_0 = 0.9050\ \text{g} + 0.1156\ \text{g} = 1.0206\ \text{g}\ ^{238}\text{U orginally present}$$

we then solve the decay equation for the elapsed time t:

$$\frac{m_t}{m_0} = \exp(-kt) = \exp\left[-\left(\frac{\ln 2}{t_{1/2}}\right)t\right]$$

$$\ln\left(\frac{m_t}{m_0}\right) = -\left(\frac{\ln 2}{t_{1/2}}\right)t$$

$$t = -\left(\frac{t_{1/2}}{\ln 2}\right)\ln\left(\frac{m_t}{m_0}\right) = -\left(\frac{4.5\times 10^9\ \text{y}}{0.6931}\right)\ln\left(\frac{0.9050\ \text{g}}{1.0206\ \text{g}}\right) = 7.8\times 10^8\ \text{y}$$

See also Example 21-7.

Three miscellaneous problems close the chapter, and the body of the book as well.

44. Quarks and antiquarks are assigned charges of either $\pm\frac{1}{3}$ or $\pm\frac{2}{3}$:

	BARYON NUMBER	CHARGE
up (u)	$\frac{1}{3}$	$\frac{2}{3}$
anti-up ($\tilde{u}$)	$-\frac{1}{3}$	$-\frac{2}{3}$
down (d)	$\frac{1}{3}$	$-\frac{1}{3}$
anti-down ($\tilde{d}$)	$-\frac{1}{3}$	$\frac{1}{3}$

Mesons are constructed from one quark and one antiquark. Neutrons and protons are constructed from three quarks:

	PARTICLE	QUARKS	BARYON NUMBER	CHARGE
(a)	π^+	$u\tilde{d}$	$\frac{1}{3}-\frac{1}{3}=0$	$\frac{2}{3}+\frac{1}{3}=1$
(b)	π^0	$u\tilde{u}$	$\frac{1}{3}-\frac{1}{3}=0$	$\frac{2}{3}-\frac{2}{3}=0$
		$d\tilde{d}$	$\frac{1}{3}-\frac{1}{3}=0$	$-\frac{1}{3}+\frac{1}{3}=0$
(c)	π^-	$\tilde{u}d$	$-\frac{1}{3}+\frac{1}{3}=0$	$-\frac{2}{3}-\frac{1}{3}=-1$
(d)	$\tilde{n}$	$\tilde{u}\tilde{d}\tilde{d}$	$-\frac{1}{3}-\frac{1}{3}-\frac{1}{3}=-1$	$-\frac{2}{3}+\frac{1}{3}+\frac{1}{3}=0$

See Section 21-6 (especially pages 789–792) and the QUESTION/ANSWER dialogue on page R21.20.

45. The ratio of a nuclear radius to an atomic radius is approximately 1 part in 100,000. The corresponding volumes, proportional to (radius)^3, differ even further:

$$\text{Volume} = \tfrac{4}{3}\pi(\text{radius})^3$$

(a) Taking $r = 1.2 \times 10^{-15}$ m, we evaluate the expression

$$R = rA^{1/3}$$

for mass numbers A = 1, 12, and 238:

NUCLEUS	A	$A^{1/3}$	R (m)
^{1}H	1	1.00	1.2×10^{-15}
^{12}C	12	2.29	2.7×10^{-15}
^{238}U	238	6.20	7.4×10^{-15}

(b) Note that the number density (nucleons per cubic meter) is independent of A:

$$\text{Density} = \frac{\text{number of nucleons}}{\text{volume}} = \frac{A}{\frac{4}{3}\pi R^3} = \frac{A}{\frac{4}{3}\pi\left(rA^{1/3}\right)^3} = \frac{A}{\frac{4}{3}\pi r^3 A}$$

$$= \frac{3}{4\pi r^3} \qquad (\text{where } r = 1.2 \times 10^{-15}\ \text{m})$$

$$= \frac{3}{4\pi\left(1.2 \times 10^{-15}\ \text{m}\right)^3}$$

$$= 1.38 \times 10^{44}\ \text{nucleons m}^{-3}$$

To express this density in g cm^{-3}, we assume that the mass of each nucleon is approximately 1.66×10^{-24} g:

$$\text{Density} = \frac{1.38 \times 10^{44}\ \text{nucleons}}{\text{m}^3} \times \frac{1.66 \times 10^{-24}\ \text{g}}{\text{nucleon}} \times \left(\frac{1\ \text{m}}{100\ \text{cm}}\right)^3 = 2.3 \times 10^{14}\ \text{g cm}^{-3}$$

(c) Weight of nuclear material packed into 1 cm^3:

$$\frac{2.3 \times 10^{14}\ \text{g}}{\text{cm}^3} \times \frac{1\ \text{kg}}{1000\ \text{g}} \times \frac{2.2\ \text{lb}}{\text{kg}} \times \frac{1\ \text{ton}}{2000\ \text{lb}} \times 1\ \text{cm}^3 = 2.5 \times 10^8\ \text{tons}$$

46. A continuation of Exercise 45.

(a) See the preceding exercise, where we showed that the average density of any nucleus is 1.38×10^{44} nucleons m^{-3} (independent of the mass number A):

$$\text{Density} = \frac{\text{number of nucleons}}{\text{volume}} = \frac{A}{\frac{4}{3}\pi R^3} = \frac{A}{\frac{4}{3}\pi\left(rA^{1/3}\right)^3} = \frac{A}{\frac{4}{3}\pi r^3 A}$$

$$= \frac{3}{4\pi r^3} \qquad (\text{where } r = 1.2 \times 10^{-15} \text{ m})$$

$$= \frac{3}{4\pi\left(1.2 \times 10^{-15} \text{ m}\right)^3}$$

$$= 1.38 \times 10^{44} \text{ nucleons m}^{-3}$$

The average volume per nucleon, the same for all nuclei, is equal to the reciprocal of the density:

$$\text{Volume per nucleon} = \frac{1}{\text{density}} = \frac{4\pi r^3}{3}$$

(b) Nearly 1.4×10^{44} nucleons can be packed into one cubic meter, as demonstrated above. The average volume per nucleon, a minute quantity, follows from the reciprocal of this enormously high nucleonic density:

$$\frac{1 \text{ m}^3}{1.38 \times 10^{44} \text{ nucleons}} = \frac{7.2 \times 10^{-45} \text{ m}^3}{\text{nucleon}}$$

(c) Attractive only at distances between $\approx 1 \times 10^{-15}$ m and $\approx 2 \times 10^{-15}$ m (approximately the radius of a nucleon), the strong force ensures that the nuclear particles are packed as tightly as possible—almost touching. The volume allowed for each nucleon remains roughly constant as the number of particles increases.