



***EROI at different analysis boundaries
and
Net energy (services) long-term trend of UK***

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Two topics on net energy measures

- **Gross and Net Energy Ratios (GER, NER) of economy as system boundary expands**
 - Mimic Costanza and Herendeen (1980, 1984) to calculate energy intensities
 - Use energy intensities to calculate GER and NER of economy
- **Energy Intensity Ratio (EIR) of long-term time series of UK energy and energy services**
 - 1500 to 2000
 - EIR is proxy for net energy ratio ($= \text{EROI} - 1$)
 - Roger Fouquet (2008) *Heat, Power, and Light*





GER and NER of energy as boundary of analysis expands

Net energy ratios can be calculated based on energy intensities, ε

$$\vec{\varepsilon}X + \vec{E}_{earth} = \vec{\varepsilon}\hat{X}$$

such that

$$\vec{\varepsilon} = \vec{E}_{earth} \left(\hat{X} - X \right)^{-1}$$

- **X** = transactions matrix
 - can have units of energy for energy sectors
- **Xhat** = diagonal of gross production each sector
- **ε** = vector of energy intensities
 - “Gross energy extracted/Net energy output”
- **E_{earth}** = gross energy extraction



Net energy ratios can be calculated based on energy intensities, ε

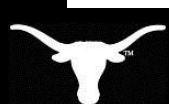
- With ε = “Gross energy extracted/Net energy output”

Gross Energy Ratio (GER)

$$EROI = GER = \frac{E_{out}}{E_{in}} = \frac{\varepsilon}{\varepsilon - 1}$$

Net Energy Ratio (NER)

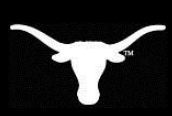
$$NER = \frac{E_{out} - E_{in}}{E_{in}} = \frac{1}{\varepsilon - 1} = GER - 1$$



Assume process information with oil, NG, and “other” sectors

$$E_{\text{earth}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

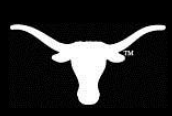
$$X = \begin{bmatrix} 0 & 0 & 1.0 & 0 & 0 \\ 0 & 0 & 0 & 1.0 & 0 \\ 0.02 & 0.07 & 0 & 0 & 0.02 \\ 0.05 & 0 & 0.06 & 0 & 0.08 \\ 0.014 & 0.013 & 0.006 & 0.005 & 0 \end{bmatrix} \begin{array}{l} \text{Oil extraction} \\ \text{NG extraction} \\ \text{Oil refining} \\ \text{NG processing} \\ \text{“Other”} \end{array}$$



Assume process information with oil, NG, and “other” sectors

$$\hat{X} = \begin{bmatrix} E_{\text{Gross oil}} & 0 & 0 & 0 & 0 \\ 0 & E_{\text{Gross NG}} & 0 & 0 & 0 \\ 0 & 0 & E_{\text{Gross refined oil}} & 0 & 0 \\ 0 & 0 & 0 & E_{\text{Gross distributed NG}} & 0 \\ 0 & 0 & 0 & 0 & E_{\text{Gross other}} \end{bmatrix}$$

$$\hat{X} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0.95 & 0 & 0 \\ 0 & 0 & 0 & 0.97 & 0 \\ 0 & 0 & 0 & 0 & 0.038 \end{bmatrix}$$

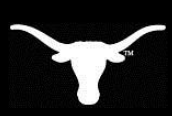


Each energy intensity represents a slightly different interpretation for net energy

$$\vec{\varepsilon} = E_{\text{earth}}(\hat{X} - X)^{-1}$$

$$\vec{\varepsilon} = \begin{bmatrix} \frac{E_{\text{Gross oil}}}{E_{\text{Net oil}}} & \frac{E_{\text{Gross oil}}}{E_{\text{Net NG}}} & \frac{E_{\text{Gross oil}}}{E_{\text{Net refined oil}}} & \frac{E_{\text{Gross oil}}}{E_{\text{Net distributed NG}}} & \frac{E_{\text{Gross oil}}}{E_{\text{Net 'other'}}} \\ \frac{E_{\text{Gross NG}}}{E_{\text{Net oil}}} & \frac{E_{\text{Gross NG}}}{E_{\text{Net NG}}} & \frac{E_{\text{Gross NG}}}{E_{\text{Net refined oil}}} & \frac{E_{\text{Gross NG}}}{E_{\text{Net distributed NG}}} & \frac{E_{\text{Gross NG}}}{E_{\text{Net 'other'}}} \end{bmatrix}$$

$$\vec{\varepsilon} = \begin{bmatrix} 1.038 & 0.087 & 1.103 & 0.094 & 0.779 \\ 0.091 & 1.044 & 0.180 & 1.088 & 2.386 \end{bmatrix}$$



Each energy intensity represents a slightly different interpretation for net energy

$$\vec{\epsilon} = E_{\text{earth}} (\hat{X} - X)^{-1}$$

$$\vec{\epsilon} = \begin{bmatrix} \frac{E_{\text{Gross oil}}}{E_{\text{Net oil}}} & \frac{E_{\text{Gross oil}}}{E_{\text{Net NG}}} & \frac{E_{\text{Gross oil}}}{E_{\text{Net refined oil}}} & \frac{E_{\text{Gross oil}}}{E_{\text{Net distributed NG}}} & \frac{E_{\text{Gross oil}}}{E_{\text{Net 'other'}}} \\ \frac{E_{\text{Gross NG}}}{E_{\text{Net oil}}} & \frac{E_{\text{Gross NG}}}{E_{\text{Net NG}}} & \frac{E_{\text{Gross NG}}}{E_{\text{Net refined oil}}} & \frac{E_{\text{Gross NG}}}{E_{\text{Net distributed NG}}} & \frac{E_{\text{Gross NG}}}{E_{\text{Net 'other'}}} \end{bmatrix}$$

$$GER_{\frac{\text{Primary energy}}{\text{Net oil}}} = \frac{\frac{\epsilon_{\text{Gross total primary}}}{\text{Net oil}}}{\frac{\epsilon_{\text{Gross total primary}}}{\text{Net oil}} - 1} = \frac{\frac{E_{\text{Gross oil}}}{E_{\text{Net oil}}} + \frac{E_{\text{Gross NG}}}{E_{\text{Net oil}}}}{\frac{E_{\text{Gross oil}}}{E_{\text{Net oil}}} + \frac{E_{\text{Gross NG}}}{E_{\text{Net oil}}} - 1} = 8.75$$

$$NER_{\frac{\text{Primary energy}}{\text{Net oil}}} = \frac{1}{\frac{\epsilon_{\text{Gross total primary}}}{\text{Net oil}} - 1} = \frac{1}{\frac{E_{\text{Gross oil}}}{E_{\text{Net oil}}} + \frac{E_{\text{Gross NG}}}{E_{\text{Net oil}}} - 1} = 7.75$$



Energy return ratios (ERRs) are most meaningful when considering total gross primary energy production

ERRs considering *TOTAL*
gross primary energy
extraction

ERR	Equation	Value
GER $\frac{\text{Primary energy}}{\text{Net oil}}$	$\frac{\epsilon_{1,1} + \epsilon_{2,1}}{\epsilon_{1,1} + \epsilon_{2,1} - 1}$	8.75
NER $\frac{\text{Primary energy}}{\text{Net oil}}$	$\frac{1}{\epsilon_{1,1} + \epsilon_{2,1} - 1}$	7.75
GER $\frac{\text{Primary energy}}{\text{Net refined oil}}$	$\frac{\epsilon_{1,2} + \epsilon_{2,3}}{\epsilon_{1,3} + \epsilon_{2,3} - 1}$	4.53
NER $\frac{\text{Primary energy}}{\text{Net refined oil}}$	$\frac{1}{\epsilon_{1,3} + \epsilon_{2,3} - 1}$	3.53
GER $\frac{\text{Primary energy}}{\text{Net NG}}$	$\frac{\epsilon_{2,2} + \epsilon_{1,2}}{\epsilon_{2,2} + \epsilon_{1,2} - 1}$	8.64
NER $\frac{\text{Primary energy}}{\text{Net NG}}$	$\frac{1}{\epsilon_{2,2} + \epsilon_{1,2} - 1}$	7.64
GER $\frac{\text{Primary energy}}{\text{Net distributed NG}}$	$\frac{\epsilon_{2,4} + \epsilon_{1,4}}{\epsilon_{2,4} + \epsilon_{1,4} - 1}$	6.49
NER $\frac{\text{Primary energy}}{\text{Net distributed NG}}$	$\frac{1}{\epsilon_{2,4} + \epsilon_{1,4} - 1}$	5.49

ERRs considering *ONE*
type gross primary energy
extraction

ERR	Equation	Value
GER $\frac{\text{Gross oil}}{\text{Net oil}}$	$\frac{\epsilon_{1,1}}{\epsilon_{1,1} + \epsilon_{2,1} - 1}$	8.04
NER $\frac{\text{Gross oil}}{\text{Net oil}}$	$\frac{1}{\epsilon_{1,1} + \epsilon_{2,1} - 1}$	7.75
GER $\frac{\text{Gross oil}}{\text{Net refined oil}}$	$\frac{\epsilon_{1,3}}{\epsilon_{1,3} + \epsilon_{2,3} - 1}$	3.90
NER $\frac{\text{Gross oil}}{\text{Net refined oil}}$	$\frac{1}{\epsilon_{1,3} + \epsilon_{2,3} - 1}$	3.53
GER $\frac{\text{Gross NG}}{\text{Net NG}}$	$\frac{\epsilon_{2,2}}{\epsilon_{2,2} + \epsilon_{1,2} - 1}$	7.97
NER $\frac{\text{Gross NG}}{\text{Net NG}}$	$\frac{1}{\epsilon_{2,2} + \epsilon_{1,2} - 1}$	7.64
GER $\frac{\text{Gross NG}}{\text{Net distributed NG}}$	$\frac{\epsilon_{2,4}}{\epsilon_{2,4} + \epsilon_{1,4} - 1}$	5.97
NER $\frac{\text{Gross NG}}{\text{Net distributed NG}}$	$\frac{1}{\epsilon_{2,4} + \epsilon_{1,4} - 1}$	5.49

The limiting case of $EROI = 1$ ($GER = 1$) is the same as saying $GDP = 0$

- Previous GER and NERs assume non-zero demand

$$\vec{d} = \begin{bmatrix} 0 \\ 0 \\ 0.84 \\ 0.78 \\ 0 \end{bmatrix}$$

- At zero demand ...
 - Energy intensities are infinite
 - $GER = 1$ and $NER = 0$ for each energy resource
 - Equivalent to saying $GDP = 0$

$$\vec{d} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$



Think about expanding boundary from normal economic I-O formulation

- Same concept as Costanza and Herendeen (1980, 1984)

Table 5. Simplified I-O matrix and economic flows for a 2-sector economy.

	Energy Sector 1	'Other' Sectors 2	Final Demand		Total Output
Energy Sector 1	x_{11}	x_{12}	g_1	c_1	x_1
'Other' Sectors 2	x_{21}	x_{22}	g_2	c_2	x_2
Taxes	t_1	t_2	t_G	t_C	T
Wages	l_1	l_2	l_G	l_C	L
Total Outlays	xx_1	xx_2	GG	CC	Y

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Imagine a simple example 2-sector economy (1 energy sector, 1 'other' sector)

Table 6. Example values for ERR calculation of expanding boundary

		Energy Sector	'Other' Sectors	Final Demand	Total Output	Units
		1	2			
Energy Sector	1	1	2	1	5	energy
'Other' Sectors	2	1	5	1	10	any
Taxes		1.2	5.44	0	0	any
Wages		l_1	l_2	l_F	l_G	any

Table 7. Example values for ERR calculation of expanding boundary

System Boundary	$\epsilon_{1,1}$	$\epsilon_{1,2}$	$\epsilon_{1,3}$	GER	NER
2x2	1.15	0.19	–	7.71	6.71
3x3	1.20	0.30	0.23	6.08	5.08
4x4	∞	∞	∞	1	0

$$E_{\text{earth}} = \begin{bmatrix} 9 & 0 \end{bmatrix}$$

$$\epsilon_{1,1} = E_{\text{gross out}} / E_{\text{net}}$$



Finding the ratio of energy price to 'other' price, you can solve for ERRs from ε in units of 'energy/\$'

	$\varepsilon_{1,1} = E_{\text{gross}}/E_{\text{net}}$				
System Boundary	$\varepsilon_{1,1}$	$\varepsilon_{1,2}$	$\varepsilon_{1,3}$	GER	NER
2x2	1.15	0.19	—	7.71	6.71
3x3	1.20	0.30	0.23	6.08	5.08
4x4	∞	∞	∞	1	0

$$\varepsilon_{1,1} = E_{\text{gross out}}/E_{\text{net}}$$

$$= E_{\text{gross out}}/\$_{\text{net}}$$

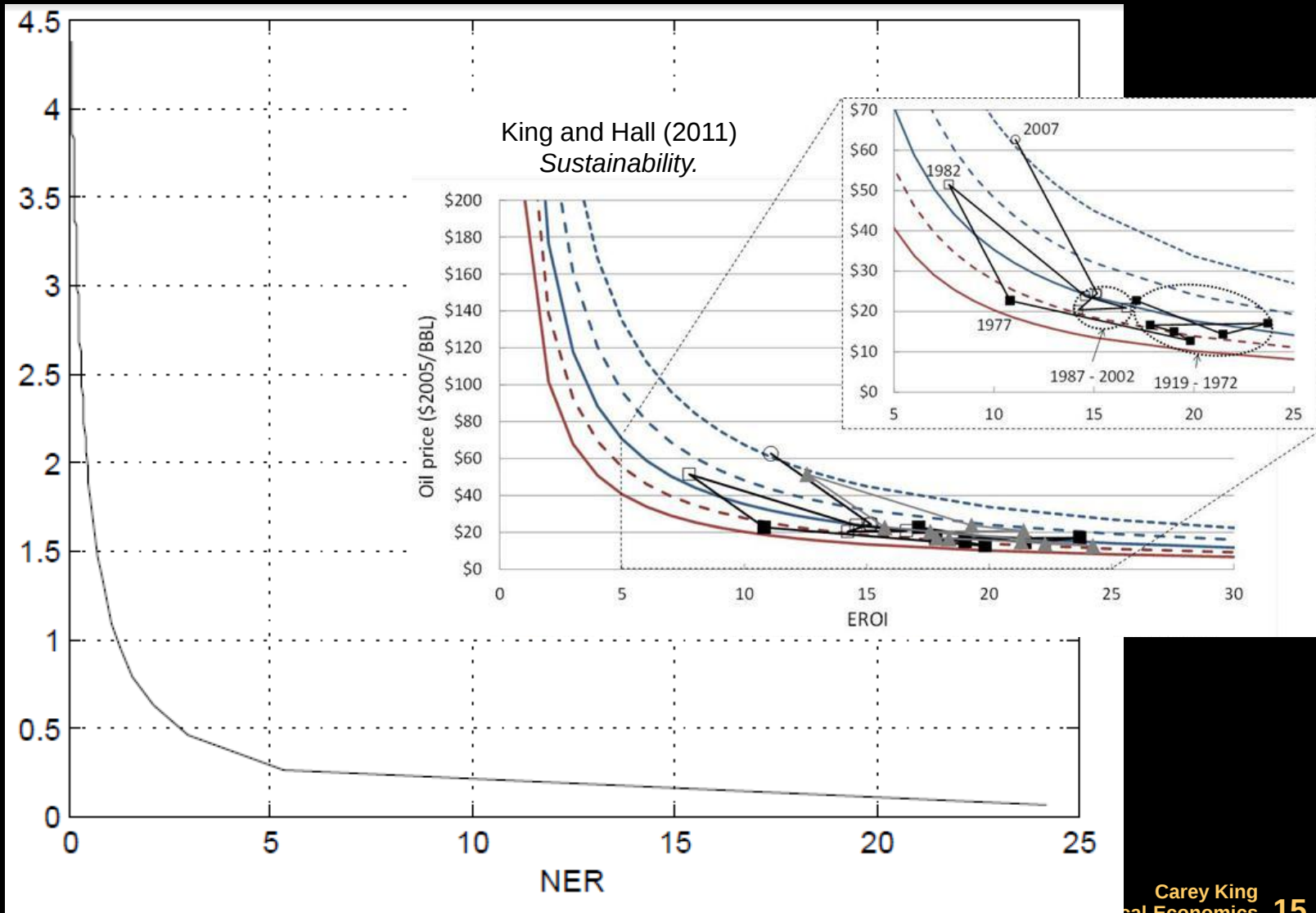
If $p_{\text{energy}} = 1$ and
 $p_{\text{other}} = 4.95$ (2x2)
 $p_{\text{other}} = 5.54$ (3x3)

System Boundary (or matrix size)	$\varepsilon_{1,1} = E_{\text{gross}}/\$_{\text{net}}$				GER	NER
	$\varepsilon_{1,1}$	$\varepsilon_{1,2}$	$\varepsilon_{1,3}$	$\varepsilon_{\text{Total Economy}}$		
2x2	1.1489	0.0397	—	0.1472	7.71	6.71
3x3	1.1965	0.0610	0.2255	0.1652	6.08	5.08
4x4	∞	∞	∞	∞	1	0

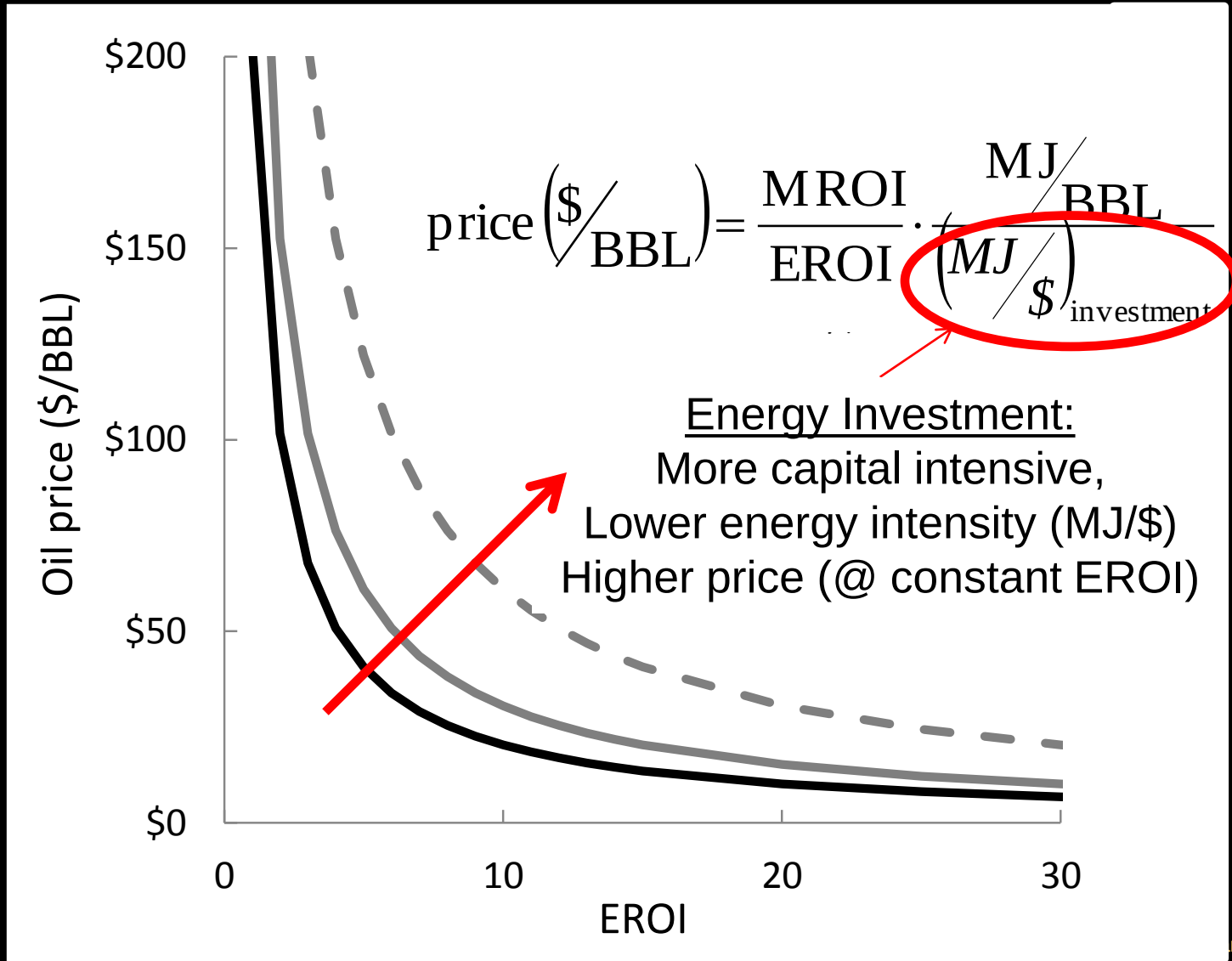


Assuming a constant demand, we can draw price vs. NER (or GER) curve

price_{energy} / price_{other}



Capital intensive energy is more expensive at equal EROI



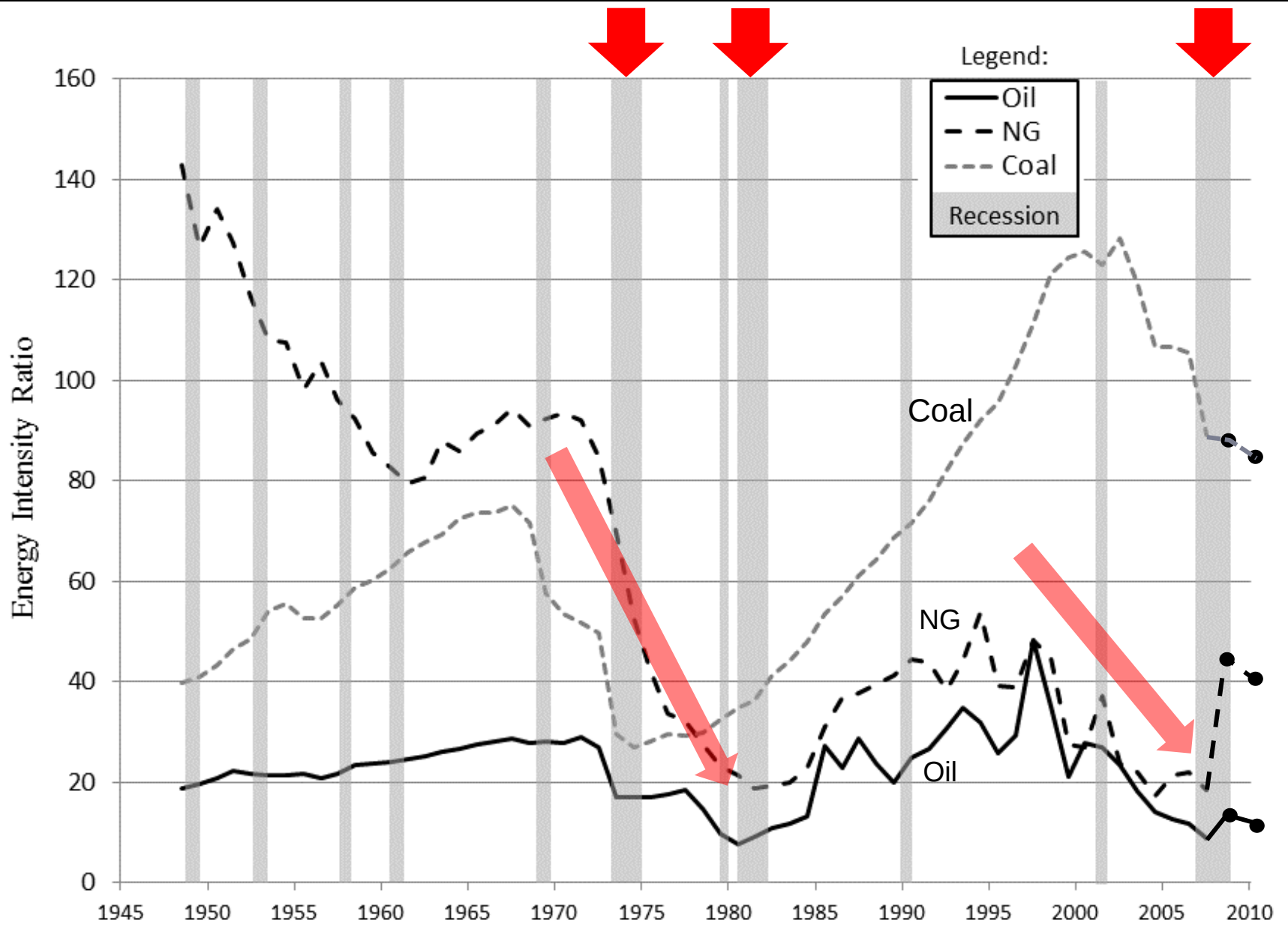


***Energy Intensity Ratio (proxy EROI) of UK
time series: 1500-2000***

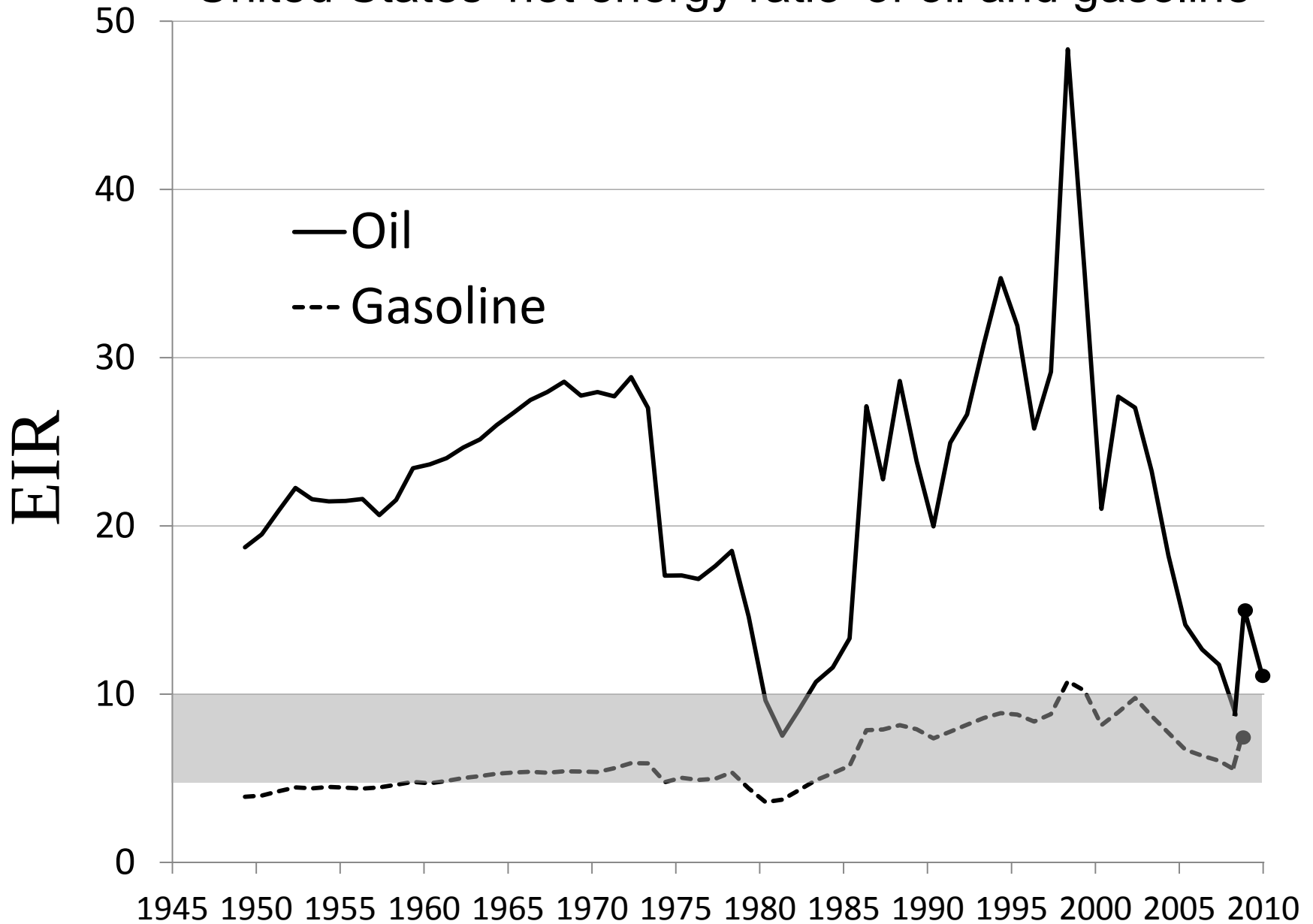
The Energy Intensity Ratio (EIR): a NER proxy using energy prices

- $EIR = (\text{"Btu/\$"} \text{ of fuel}) / (\text{"Btu/\$"} \text{ of economy}) \sim E_{out}/E_{in}$
 - “What is the cost of providing energy to the economy relative to the average productivity from consuming energy?”
- Example (using US 2008 values in \$2008):
 - Oil at \$100/BBL $\rightarrow$ 57,000 Btu/\$2008
 - US economy: 6,960 Btu/\$2008 GDP
 - $EIR_{oil, price} = 57,000/6,960 = 8.3$

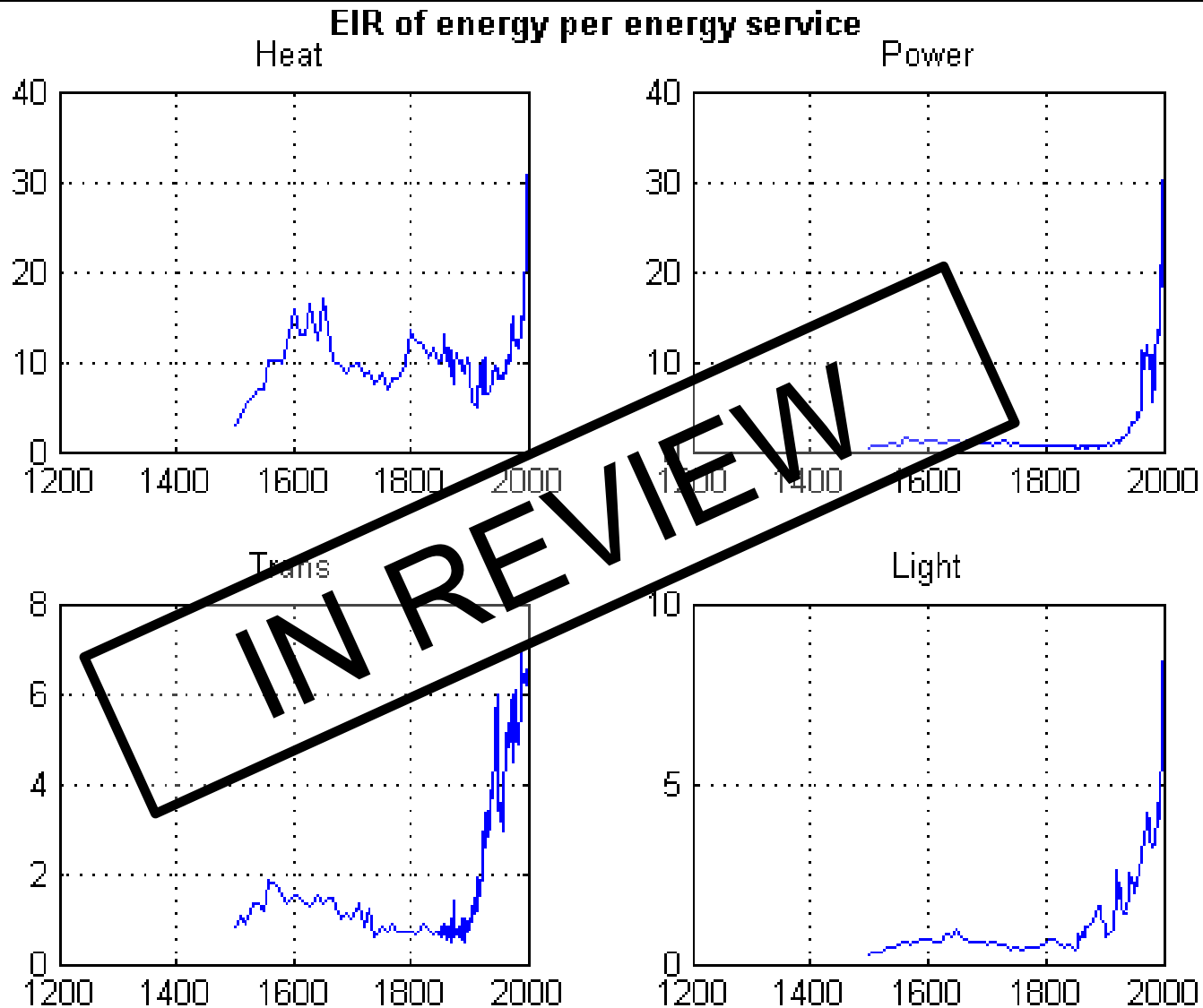




United States 'net energy ratio' of oil and gasoline



UK EIR based upon energy purchased per service

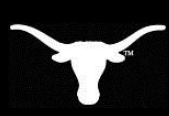


Data from Fouquet, R. (2008) *Heat, Power, and Light: Revolutions in Energy Services*.

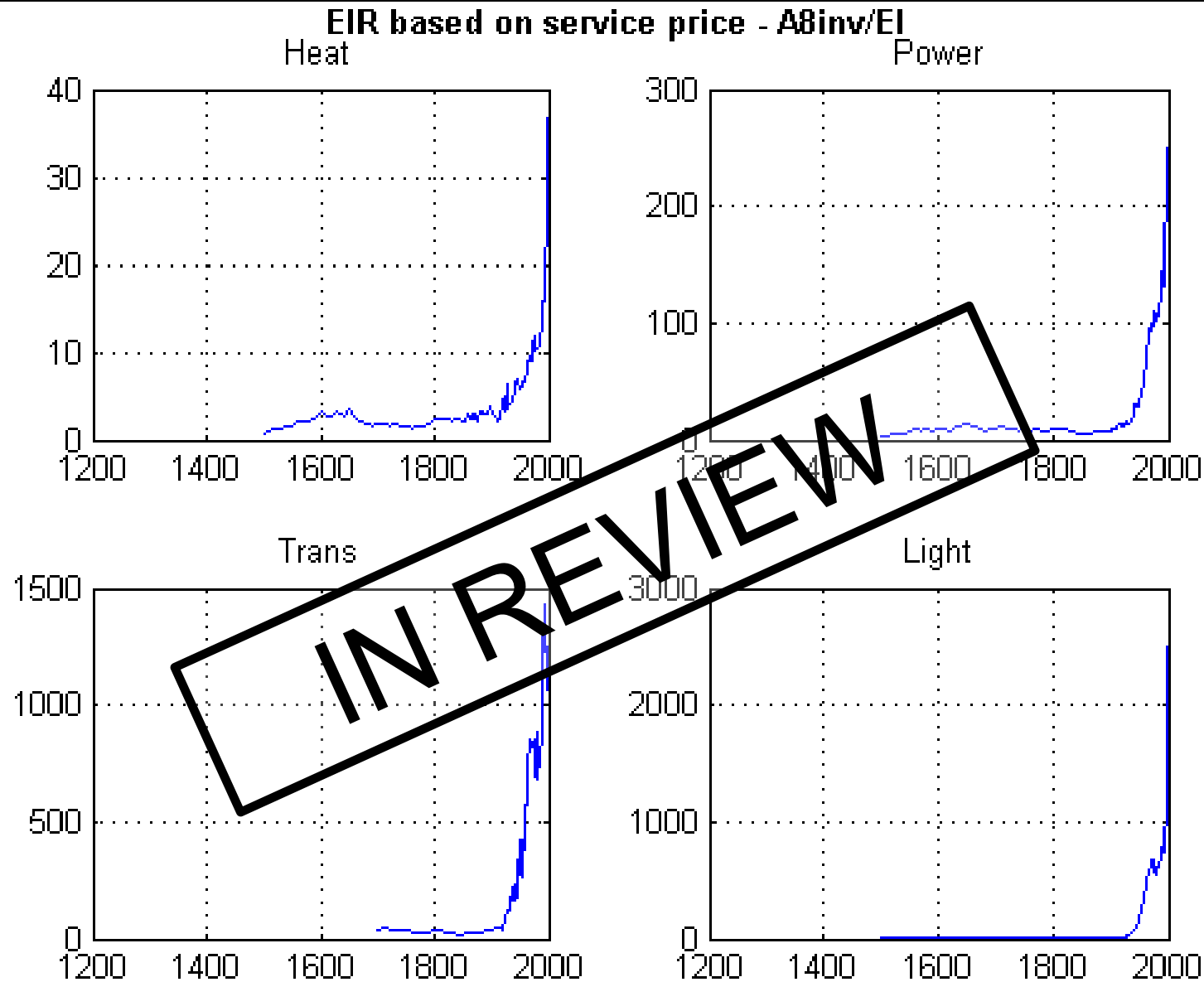


UK EIR based upon energy service itself

- **Transportation = [passenger·km/\$] / [energy/GDP]**
- **Power = [energy/\$] / [energy/GDP]**
 - Power ~ useful work (animal work, electricity)
- **Heat = [energy/\$] / [energy/GDP]**
- **Light = [lumens/\$] / [energy/GDP]**

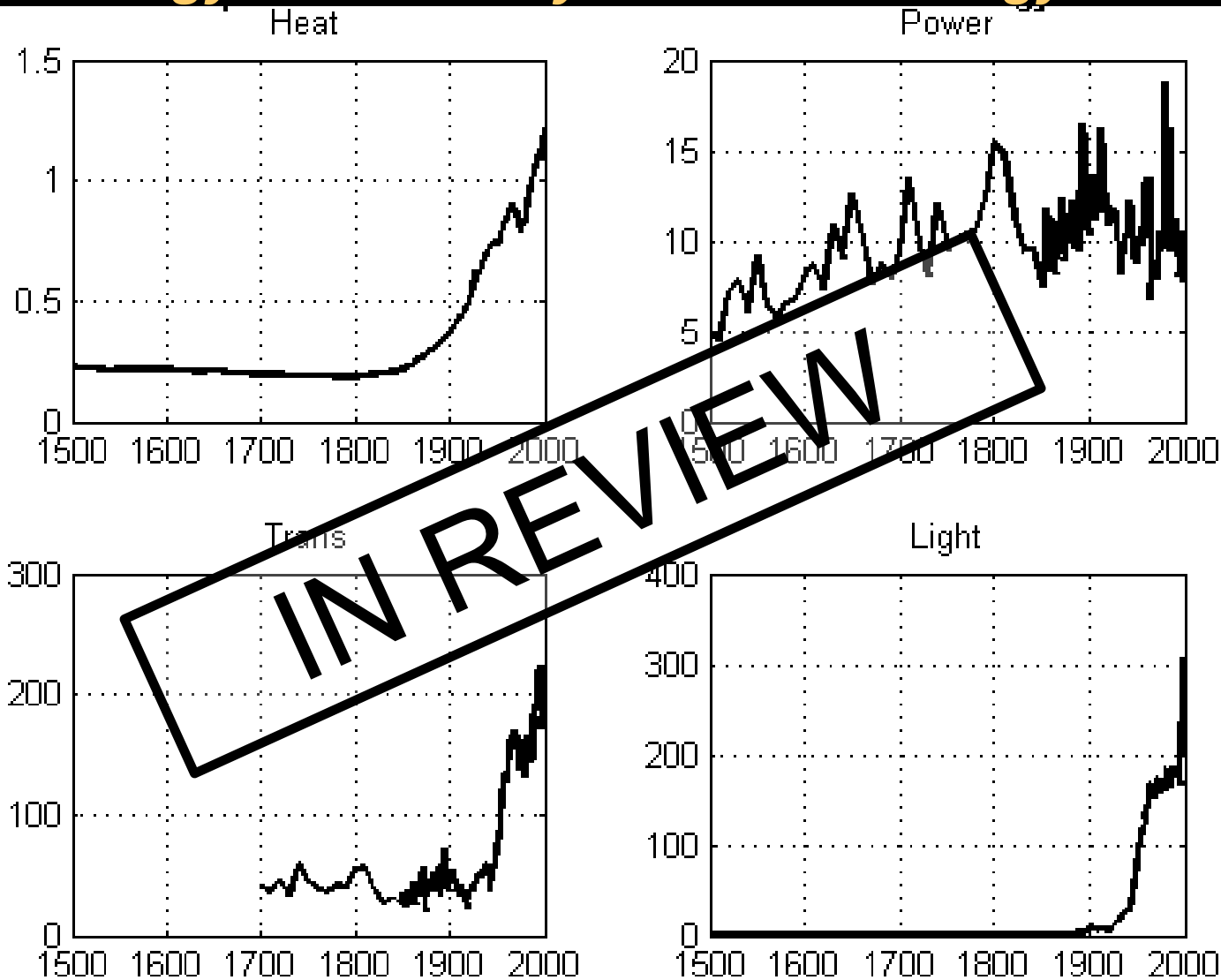


UK EIR based upon energy service itself



Data from Fouquet, R. (2008) *Heat, Power, and Light: Revolutions in Energy Services*.

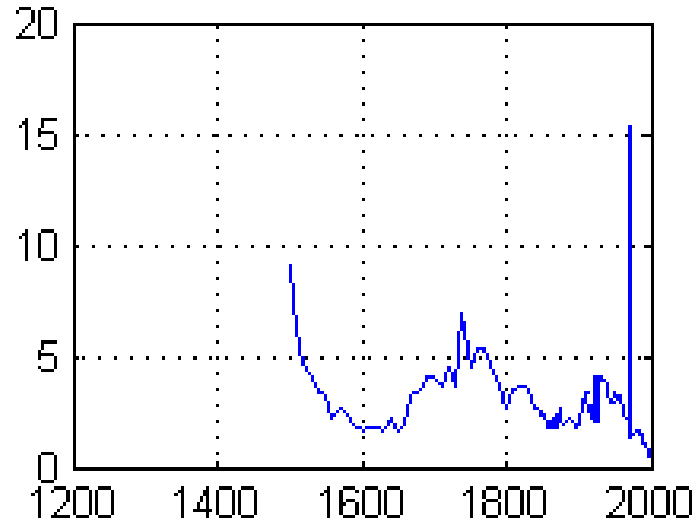
$EIR_{\text{service}} / EIR_{\text{energy}} = (\text{£/energy}) / (\text{£/service})$
increasing trend indicates benefits from investments in end-use technology & efficiency relative to energy investment



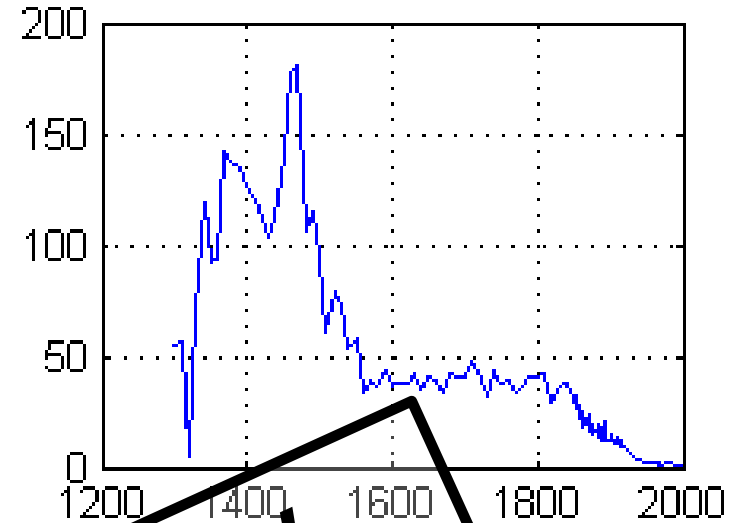
Data from Fouquet, R. (2008) *Heat, Power, and Light: Revolutions in Energy Services*.

Percent GDP Spent On Energy for Service

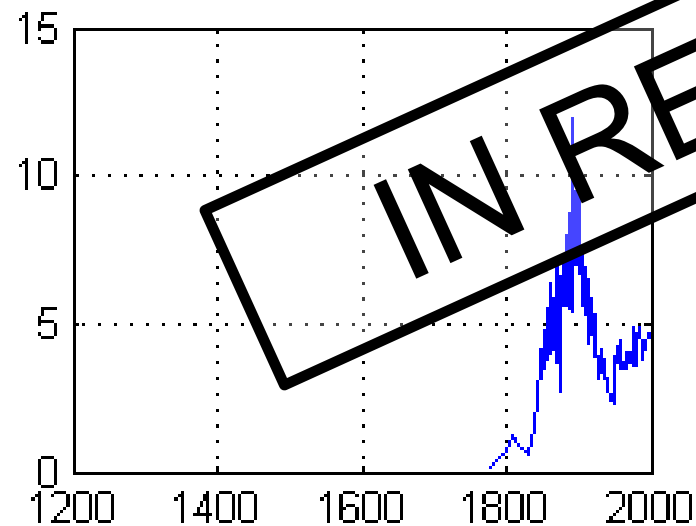
Heat



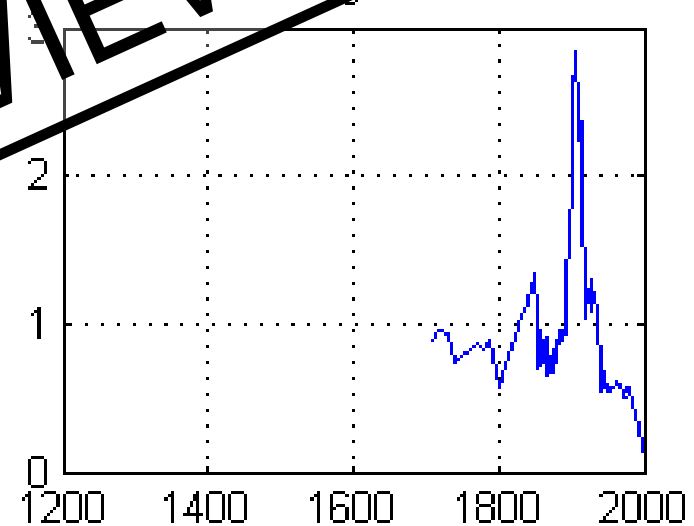
Power



Trans



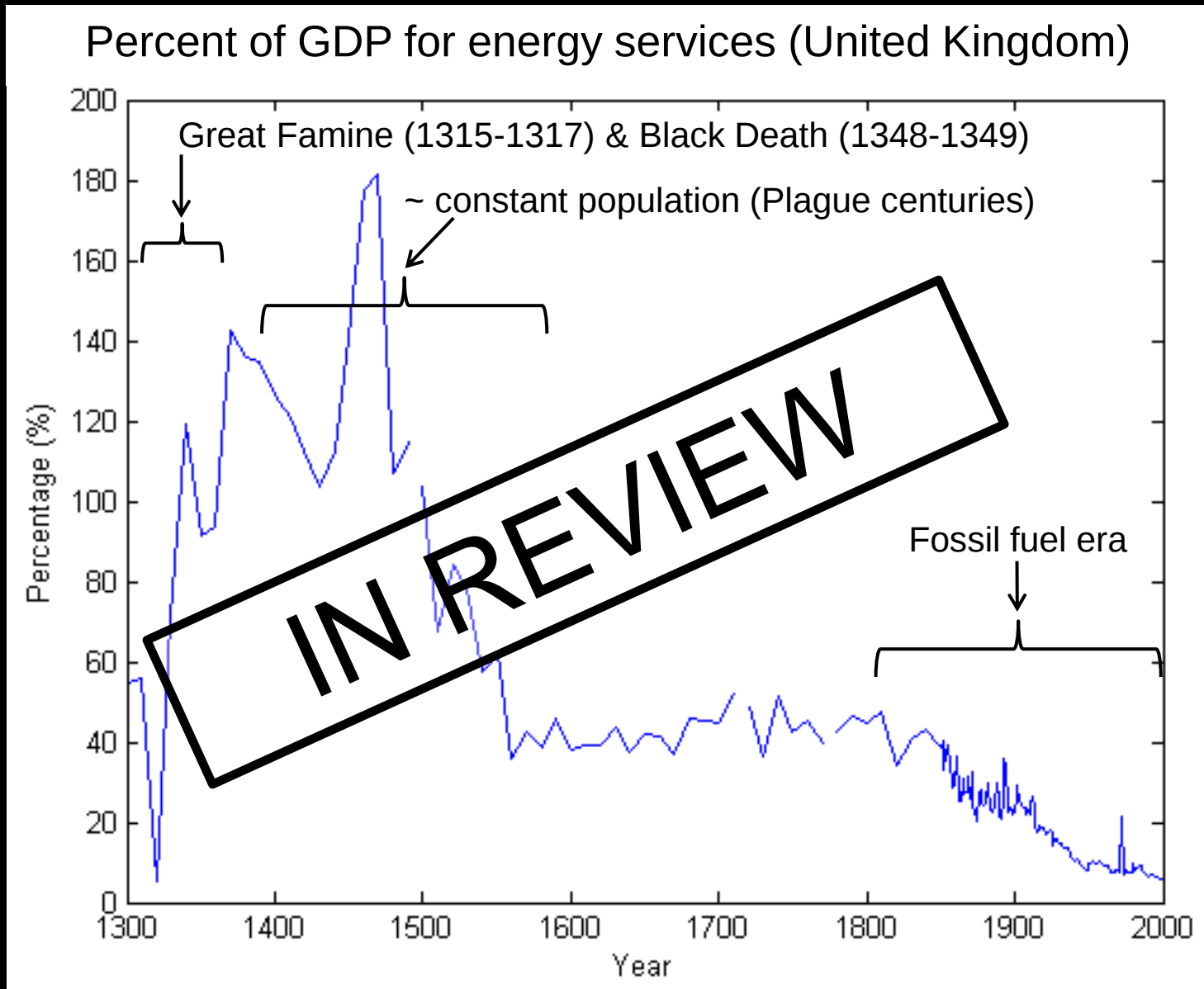
Light



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Data from Fouquet, R. (2008) *Heat, Power, and Light: Revolutions in Energy Services*.

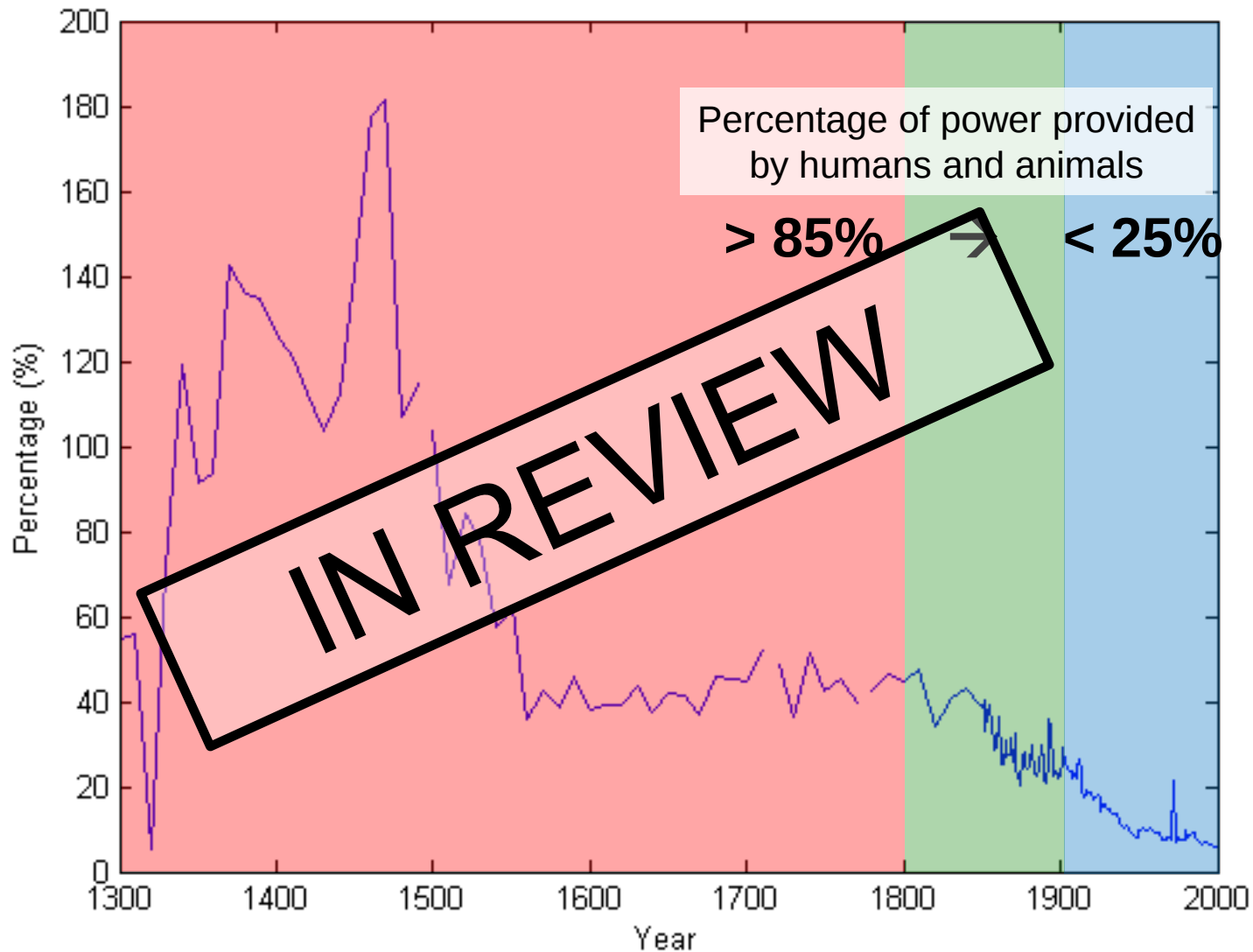
Energy and food expenditures have decreased for centuries



Data from R. Fouquet (2008) *Heat, Power and Light: Revolutions in Energy Services*.

Food was the early energy supply ... for human and animal power

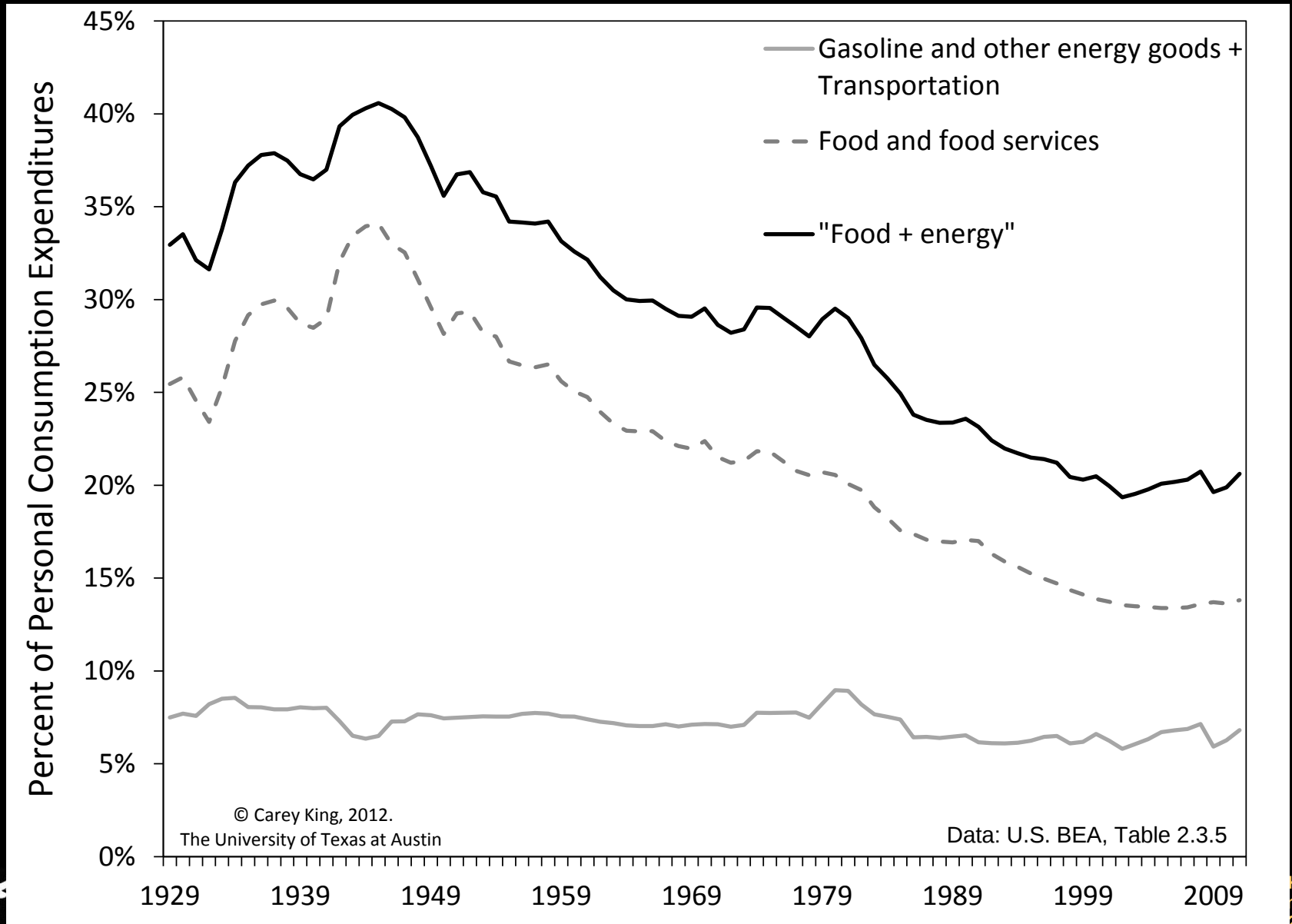
Percent of GDP for energy services (United Kingdom)



Data from R. Fouquet (2008) *Heat, Power and Light: Revolutions in Energy Services*.



‘Food + energy’ expenditures are relevant for long-term analysis





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