A METHOD OF CALCULATING THE THICKNESSES OF THE ICE-SHEETS

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ICE-SHEETS, such as those of the Pleistocene epoch and those that now cover Greenland and the Antarctic continent, are in a state of quasi-static equilibrium in which the tendency of the ice to spread laterally and to flow downhill under its own weight is just balanced by the inward shear force exerted by the rock floor. By estimating the magnitude of this shear force, it is possible to make an approximate calculation of the thicknesses of the ice.

We may consider first a slab of ice of large extent and of uniform thickness h resting on a plane bed of inclination α. If the stresses are the same on every cross-section perpendicular to the line of greatest slope, resolution of forces down the slope shows that the shear stress on the bed is

$$\tau = \rho g h \sin \alpha, \qquad (1)$$

where ρ is the density and g is the gravitational acceleration. We now allow the thickness and slope to vary slowly from point to point. Let the surface be of slope α in the region to be considered (taken to be of large extent compared with h), but let the bed now be of slightly different inclination. The lines of greatest slope of the bed and of the surface are not now, in general, in the same vertical plane. The shear stress on a plane drawn parallel to the surface, at a distance h from it, is still given approximately by $\rho g h \sin \alpha$ for any value of α, however small. The shear stress on the bed itself is obtained by making a small rotation of the axes of reference of the stress tensor, the small rotation being that which it would be necessary to apply to the surface to make it parallel to the bed. It follows that, since the bed is not near a plane of principal stress (it is, in fact, close to a plane of maximum shear stress), the shear stress on it is also given approximately by the above expression. Moreover, this shear stress is directed parallel, in plan view, to the line of greatest slope of the surface, which implies in turn that the direction of movement of the ice must also be parallel, in plan view, to the line of greatest slope of the surface. Formula (1) is thus approximately true for a sheet of ice resting on a bed the slope of which changes both in magnitude and direction, provided that the local values of the thickness h and surface slope α are used and that h and α do not change much in distances of order h. For an irregular bed the formula gives the shear stress averaged over areas of linear dimensions of order h.

Calculations on Alpine valley glaciers[1] show that the shear stresses on their beds are between 0·5 and 1·5 bars (1 bar = 10^6 dynes/cm.^{2}). The values are spread by such a comparatively small amount because the rate of strain in ice under a sustained stress depends upon a high power (~ 4) of the applied shear stress[2]; thus sustained shear stresses much smaller than 1 bar produce extremely small rates of strain ($\ll 1$ per year), while shear stresses much greater than this produce very much larger rates of deformation than those existing in glaciers and ice sheets. It may not be far wrong, therefore, to assume for a first calculation that τ is constant (~ 1 bar) over the floor of a moving ice-sheet. (The stipulation that the ice-sheet is moving is necessary, because in the centre of a large ice-sheet of considerable height it is possible that the precipitation is not sufficiently in excess of the wastage by evaporation and melting to keep the mass moving; in such a case a considerably smaller shear stress could exist on the bed, and the height would be determined by meteorological factors rather than in the way considered here.) The pressure at the base of an ice-sheet can reach values of several hundred bars, but, in view of the fact that in other crystalline plastic solids the yield stress is nearly independent of a superposed hydrostatic pressure, it is probable that in ice a pure hydrostatic pressure has a negligible effect upon the shear stress needed to produce a given rate of shear strain.

Equation (1) may be written as

$$h = h_0 \operatorname{cosec} \alpha,$$

or, for small α,

$$h = h_0/\alpha, \qquad (2)$$

where $h_0 = \tau/\rho g$. If $\tau = 1$ bar, $h_0 = 11$ m. We therefore have the very simple result that the thickness of a moving ice-sheet is inversely proportional to its surface slope. This result can be applied to two types of problem.

(a) *Bed known; to calculate the ice surface.*

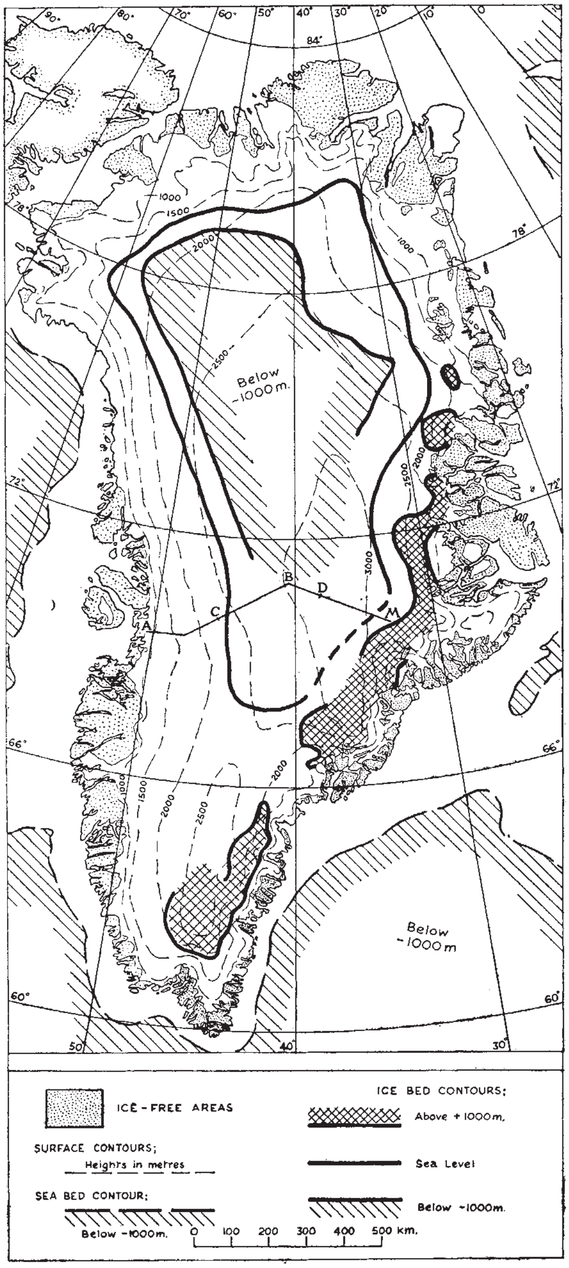
Equation (2) can be written as

$$\delta H = \frac{h_0}{h}\,\delta s, \qquad (3)$$

where δH is the increase in absolute height of the ice surface as one moves a distance δs along a line of flow, in the opposite direction to the movement of the ice. The lines of flow of the Pleistocene ice-sheets are approximately known in some places from the directions of the striations on the rocks and from other erosional evidence. Equation (3) may then be used to integrate step by step along a line of flow, starting at a point where the thickness is known. (If the integration is started at the edge of the ice-sheet, $h = 0$, the first increment in H should be taken by putting $h = \delta H/2$, so that $\delta H = \sqrt{2h_0 \delta s}$, to avoid the infinity that would arise from a straightforward application of the formula.) The successive increases in H are calculated by using for each interval the value of h found by subtracting the height of the bed from the calculated value of H, making allowance as necessary for the depression of the bed by the overlying ice. The profile so found will not be accurate very near the edge, say, within 1 km., where h is changing rapidly.

If the bed is horizontal, $\delta H = \delta h$, and the step by step integration is unnecessary. Equation (3) integrates to the parabola $h = \sqrt{2h_0 s}$, which was obtained by Hill[3] and by Orowan[4] in the special case when the lines of flow in plan view are all parallel; or it can be expressed by the relation

$$\operatorname{grad}(h^2) = 2h_0.$$



Greenland, showing the height of the bed calculated from equation (2) with $h_0 = 10 \cdot 0$ m.

Where the bed is horizontal, therefore, the surface representing h^2 as a function of position is a surface of constant maximum slope. To find the surface for an arbitrary boundary a sand-heap analogy may be used. A sand-heap is made upon a flat base the outline of which is similar to that of the floor of the ice-sheet. The height of the sand at any point then gives, with a suitable constant of proportionality, the square of the height of the ice-sheet at the corresponding point. (Photographs of sand-heaps upon various bases are given by Nádai[5].)

The French expedition in Greenland during 1948–51, led by M. Paul E. Victor, has reported[6] the results of seismic soundings on the line *AM* shown in the map; from *C* to *D*, which is the highest point on this traverse, the floor is practically horizontal and at sea-level ($\pm$ 200 m.). These measurements of the floor level, together with the measured ice thickness at *C*, have been used to integrate equation (3) with a constant value of $\tau = 0 \cdot 88$ bars ($h_0 = 10 \cdot 0$ m.), chosen to give the best fit. The integration extends over a mountain range near *A* 600 m. in height. The ice surface so calculated between *A* and *B* is everywhere within 80 m. of the observed surface; the calculated surface is parabolic from *C* to *B*.

(*b*) *Ice surface known; to calculate the bed.*

Equation (2) implies that the height of the bed of an ice-cap can be deduced from observations on its surface only. The map shows the result obtained for Greenland by using the surface contours, which cannot be expected to be very accurate, of the Danish 1938 survey map (1 : 5,000,000). τ has been assumed to be 0·88 bars everywhere. Owing to the uncertainties in τ and α, only the broad features are significant. In the north a large area of floor appears to be below sea-level. With the above value of , one calculates a maximum depth of 4,200 m. below sea-level. Even if the value of τ is reduced to 0·5 bars, the lowest point is still 1,200 m. below sea-level, and so, although the depth may not be as much as 4,200 m., the existence of a considerable depression seems probable. The seismic soundings planned in this area by the British North Greenland Expedition leaving this summer under Commander (L) C. J. W. Simpson, R.N., are awaited with interest.

I wish to thank Mr. W. M. Lomer for his helpful suggestions on the presentation of these results.

[1] Nye, J. F., *J. Glaciol.* (in the press).

[2] Glen, J. W., "Experiments on the Plastic Deformation of Ice", submitted to *J. Glaciol.*

[3] Nye, J. F., *Proc. Roy. Soc.*, A, **207**, 554 (1951).

[4] Orowan, E., *J. Glaciol.*, 1, 231 (1949).

[5] Nádai, A., "Plasticity", Chapter 19 (McGraw-Hill, 1931).

[6] Joset, A., Expéditions Polaires Françaises. Rapport prélim. de la Section Sondages Séismiques 1950, read by J. Martin at the 1951 Conf. Int. Comm. on Snow and Ice (Brussels).

CHANGES IN THE CROSS-STRIATION OF MYOFIBRILS DURING CONTRACTION INDUCED BY ADENOSINE TRIPHOSPHATE

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Many of the observations which laid the foundations of our knowledge of muscle cytology were made on the large myofibrils (sarcostyles) which can easily be teased, with fine needles, out of the fresh thoracic muscles of many insects. It is much more difficult[1] to isolate myofibrils from the muscles of vertebrate animals. For most cytological work on such muscles it has been necessary in the past to use fixed and sectioned material. This is unfortunate, because biochemists and physiologists nearly always choose mammalian or amphibian muscles for their experiments. Recently, however, Schick and Hass[2] and Perry[3] have devised methods for isolating fibrils from fresh mammalian muscle