Math 021-Z1 Summer 2014
Final Exam Review Questions:

Limits: Evaluate the limit, if it exists:

$$\lim_{x\to 6} \sqrt{x^3+3x-7}$$

$$\lim_{t\to 0} \left(\frac{1}{t} - \frac{1}{t^2+t}\right)$$

$$\lim_{x\to 1} \frac{x^2+2x+1}{x^4-1}$$

$$\lim_{x\to 1} \frac{x^2-1}{|x-1|}$$

$$\lim_{\theta\to \pi} \frac{\cos(\theta)-1}{\sin(\theta)-1}$$

$$\lim_{x\to \infty} \frac{x^3+5x+1}{2x^3-x^2+\pi}$$

$$\lim_{x\to \infty} e^{-2x}\cos(x)$$

$$\lim_{x\to \infty} \frac{x+2}{\sqrt{9x^2+1}}$$

$$\lim_{t\to -\infty} \frac{t^2+2}{t^3+t^2-1}$$

$$\lim_{t\to \infty} \arctan(x^2-x^4)$$

$$\lim_{x\to 0} \frac{x+\tan(x)}{\sin(x)}$$

$$\lim_{x\to 0} \frac{e^x-1}{x^3}$$

$$\lim_{x\to \infty} x^2 e^{-x^2}$$

$$\lim_{x\to 0} \cos(x)^{\frac{1}{x^2}}$$

Derivatives: Find the derivative of the following:

$$f(x) = x^5 + 2x^3 + \pi^2 x^3 + \ln(\pi)$$

$$f(x) = \ln(3x^5 + 2x - 7)^{5/2}$$

$$f(x) = \frac{x^3 + 2\sqrt{x} + \pi^2}{x^{3/2}}$$

$$f(x) = 3e^{\sqrt{x^3+2x+ln(6)}}$$

$$f(x) = (t^3 + e^{t^2})(3 - \sqrt{t})$$

$$f(x) = \frac{1 + \sin(2x)}{x + \cos(3x)}$$

$$f(x) = \sin(\tan(\sqrt{\sin(x)}))$$

$$f(x) = e^{\tan(x^3)} + \ln(\sin(\sqrt{x}))$$

Use Logarithmic Differentiation to find the following derivatives with respect to x:

$$y = \frac{(\sin^2 x)(\tan^4 x)}{(x^2+1)^2}$$

$$y = \sqrt{x}e^{x^2}(x^3+1)^{10}$$

Implicit Differentiation/Related Rates:
Assume y is a function of x. Find the Derivative of the following with respect to x.
For the first part, find the equation of the tangent line at the point (1,1)

$$x^3 + x^2 y + 4y^2 = \pi^2$$

$$e^{x^2 y} = x - y$$

$$\sin(x) + \cos(y) = \sin(x)\cos(y)$$

Now assume x and y are functions of t. Evaluate dy/dt for each of the following:

$$8y^3 + x^2 = 1; \quad \frac{dx}{dt} = 2, x = 2, y = 1$$

$$\frac{y^3 - 4x^2}{x^3 + 2y} = 5; \frac{dx}{dt} = 5, x = 1, y = 0$$

$$y\ln(x) = xe^y; \quad \frac{dx}{dt} = 5, x = 1, y = 0$$

Relative Extrema:

Find the intervals where the function is increasing and decreasing. Use the first derivative test to find the relative extrema.

$$f(x) = \frac{(5-9x)^{2/3}}{7} + 1$$

$$f(x) = x^2 e^{-x^2}$$

$$f(x) = \frac{x^2}{\ln(x)}$$

Now find intervals of concavity, inflection points, and relative extrema, using the second derivative test:

$$f(x) = 2x^3 - 3x^2 + 2$$

$$f(x) = 4x^3 - 6x^2 + 7$$

$$f(x) = \frac{x^5}{5} - x$$

Absolute Extrema: Find all absolute extrema of the following functions

$$f(x) = (x^2-4)^{1/3} \text{ on } [-2, 3]$$

$$f(x) = e^{\sqrt{x^2-4}} \text{ on } [2, \infty]$$

Optimization:

Given $x + y = 90$ maximize the product x^2y

Given $x + y = 105$ maximize the product xy^2

Integration: Evaluate the following integrals.

$$\int 6x^2 - 4x^{3/2} + \frac{5}{x} + \cos(2x) + e^{3x}\, dx$$

$$\int x(\sqrt{x} + \frac{\sin(3x)}{x} + \frac{5}{x^2})\, dx$$

$$\int_1^e \frac{x^2 + x + 1}{x}\, dx$$

$$\int_0^{\pi/4} \frac{1 + \cos^2(\theta)}{\cos^2(\theta)}\, d\theta$$

$$\int_0^1 \frac{4}{t^2 + 1}\, dt$$

$$\int_{-2}^2 |x + 1|\, dx$$

Substitution Rule: Evaluate the following integrals by substitution.

$$\int x \cos(x^2)\, dx$$

$$\int x^3 e^{x^4+5}\, dx$$

$$\int \cos(\theta) \sin^6(\theta)\, d\theta$$

$$\int \frac{3x}{x^2 + 1}\, dx$$

$$\int \frac{\sin(x)}{1 + \cos^2 x}\, dx$$

$$\int_0^{\pi} \sec^2\left(\frac{\theta}{4}\right)\, d\theta$$

$$\int_e^{e^4} \frac{dx}{x\sqrt{\ln(x)}}$$

$$\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx$$

Integration by Parts: Evaluate the following integrals using integration by parts:

$$\int_0^1 xe^{-x}\,dx$$

$$\int x\ln(x)\,dx$$

$$\int_0^{\pi} t\sin(3t)\,dt$$

$$\int_0^1 \frac{2x+1}{e^x}\,dx$$

$$\int \sin(t)e^t\,dt$$

Bonus: Use rapid repeated integration to solve the following integrals by parts:

$$\int x^5 e^x\,dx$$

$$\int x^4\cos(x)\,dx$$