Exam III

Math 019G

Due: April 30, 2013

- This Take-Home exam is out of 60 points.
  Answer All Questions
- ***You must show all work to receive full credit.***

Name______________________________

Problem 1 *(20pts)*: Find all relative extrema, intervals of concavity and inflection points of the following equation.

$$f(x) = x^{7/3} + x^{4/3}$$

Problem 2 *(10pts)*: Find all absolute extrema of $f(x) = e^{-\frac{x^3}{3}+4x+3}$ on the interval $[0, \infty)$

Problem 3 *(10pts)*: You are trying to build a ski rack for your car. An open rectangular box is to be made by cutting a square corner from a 8ft by 3ft piece of metal and then folding up the sides. Let x represent the side-length of the square being cut from the corners of the metal. Find the value of x that maximizes the volume of the box.
Hint: Review your notes.

Problem 4 *(10pts)*: Suppose y is a function of x (i.e. y = y(x)). Use implicit differentiation to solve the following equation for $\frac{dy}{dx}$. Express your answer with positive exponents for all variables.

$$ln(3xy) = e^{y^3} + 4x^2$$

Problem 5 *(10pts)*: A spherical snowball is placed in the sun. The sun melts the snowball so that its radius decreases at a rate of $\frac{1}{4}$ inches per hour. Find the rate of change of the volume of the snowball with respect to time at the instant the radius is 4 inches.

part a: The volume of a sphere is given as $V = \frac{4}{3}\pi r^3$. Here, both the volume and radius of the snowball are changing over time, i.e. V = V(t) and r = r(t). Use implicit differentiation to find $\dfrac{dV}{dt}$

part b: Now we want to find how much the volume is changing when the radius, r = 4 inches. The rate of change of the radius with respect to time is constantly decreasing at $\frac{1}{4}$ inches per hour (i.e. $\frac{dr}{dt} = -\frac{1}{4}$ inches per hr).

Use this information, along with your work from part a, to find $\dfrac{dV}{dt}$ given these inputs. Make sure to express your answer with the appropriate units.