Quiz 2 Practice Problems

1. Given the Preference schedule below, find the full ranking with the method of **Pairwise Comparisons**.

<table>
<thead>
<tr>
<th>Number of Voters</th>
<th>15</th>
<th>13</th>
<th>10</th>
<th>3</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>A</td>
<td>C</td>
<td>D</td>
<td>B</td>
<td>A</td>
</tr>
<tr>
<td>2nd</td>
<td>B</td>
<td>B</td>
<td>B</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>3rd</td>
<td>C</td>
<td>A</td>
<td>C</td>
<td>D</td>
<td>D</td>
</tr>
<tr>
<td>4th</td>
<td>D</td>
<td>D</td>
<td>A</td>
<td>A</td>
<td>B</td>
</tr>
</tbody>
</table>

2. In an election with 7 candidates, using the method of **Pairwise Comparisons**, how many comparisons would you have to make?
   (ex. In the above example you had 6)
   *Hint: Use the formula from class!*

3. State in your own words what Arrow’s Impossibility Theorem means for voting systems (The big picture).

4. A company has 30 shares and a board of directors of 5 people. \( P_1 \) has 12 shares, \( P_2 \) has 7 shares, \( P_3 \) has 4 shares, \( P_4 \) has 4 shares, and \( P_5 \) has 3 shares. The quota is \( \frac{3}{4} \) of the total shares.

   a) Put the following scenarios into standard notation \([q: w_1, w_2, ..., w_n]\

   b) Does any player have **veto power**?

   c) For what quotas, \( q \), is the coalition \( \{P_2, P_3, P_4, P_5\} \) a winning coalition?
   *Hint: Remember in general, \( \frac{V}{2} < q \leq V \)
4. Consider the weighted voting system \([q: 7, 6, 5, 3, 3, 2]\).

   a) What are the bounds for the quota, \(q\), if we don't want Anarchy or Gridlock?
   
   *Hint: Remember the bounds discussed in class.*

   b) What will the value of the quota be if \(at least\ 2/3\) of the votes are required to pass the motion?

   c) What will the value of the quota be if \(more than\ \frac{3}{4}\) of the votes are required to pass the motion?

5. Consider the weighted voting system \([q: 10, 9, 8, 5, 2]\).

   a) Write down all winning coalitions for \(q = 18\).

   b) For \(q = 17\), are any of the players in the coalition \(\{P_1, P_3, P_4\}\) critical? If so, which?

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**THE FOLLOWING QUESTIONS WILL NOT BE ON THIS QUIZ, BUT IT IS GOOD PRACTICE!**

6. Consider the weighted voting system: \([16: 7: 5, 5, 4]\)

   a) Write down all coalitions.

   b) Write down all the winning coalitions.

   c) Underline the Critical Players in each coalition.

   d) Find the critical count for each player and compute \(T\), the total.

   e) Compute the Banzhaf Power distribution for this system.

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5. For the weighted voting system, \([11: 6, 5, 4, 4]\), compute the Banzhaf Power distribution.