Chapter 3 - Normal Distribution

3.1 a. Original data:

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</table>

b. To convert the distribution to a distribution of $X - \mu$, subtract $\mu = 4$ from each score:

-3  -2  -2  -1  -1  0  0  0  0  1  1  1  2  2  3

c. To complete the conversion to $z$, divide each score by $\sigma = 1.63$:

-1.84  -1.23  -1.23  -0.61  -0.61  -0.61  0  0  0  
0.61  0.61  0.61  1.23  1.23  1.84

3.3 Errors counting shoppers in a major department store:

$$z = \frac{X - \mu}{\sigma}$$

a. $\frac{960 - 975}{15} = -\frac{15}{15} = -1$  between -1 and $\mu$ lie .3413

$\frac{990 - 975}{15} = +\frac{15}{15} = +1$  between +1 and $\mu$ lie .3413

Therefore between 960 and 990 are found approximately 68% of the scores.

b. 975 = $\mu$; therefore 50% of the scores lie below 975.

c.  .5000 lie below 975
   .3413 lie between 975 and 990
   .8413 (or 84%) lie below 990
3.5 The supervisor's count of shoppers:

\[ z = \frac{X - \mu}{\sigma} = \frac{950 - 975}{15} = -1.67 \]

X to ±1.67 = 2(0.0475) = .095; therefore 9.5% of the time scores will be at least this extreme.

3.7 They would be equal when the two distributions have the same standard deviation.

3.9 Next year's salary raises:

a. 

\[ z = \frac{X - \mu}{\sigma} = -1.2817 = \frac{X - 2000}{400} \]

$2512.68 = X$

10% of the faculty will have a raise equal to or greater than $2512.68.

b. 

\[ z = \frac{X - \mu}{\sigma} = -1.645 = \frac{X - 2000}{400} \]

$1342 = X$

The 5% of the faculty who haven't done anything useful in years will receive no more than $1342 each, and probably don't deserve that much.

3.11 Transforming scores on diagnostic test for language problems:

\[ X_1 = \text{original scores} \quad \mu_1 = 48 \quad \sigma_1 = 7 \]
\[ X_2 = \text{transformed scores} \quad \mu_2 = 80 \quad \sigma_2 = 10 \]
\[ \sigma_2 = \sigma_1 / C \]
\[ 10 = 7 / C \]
\[ C = 0.7 \]

Therefore to transform the original standard deviation from 7 to 10, we need to divide the original scores by .7. However dividing the original scores by .7 divides their mean by .7.

\[ \bar{X}_2 = \bar{X}_1 / .7 = 48 / .7 = 68.57 \]

We want to raise the mean to 80. \( 80 - 68.57 = 11.43 \). Therefore we need to add 11.43 to each score.

\[ X_2 = X_1 / 0.7 + 11.43 \]  [This formula summarizes the whole process.]

3.13 October 1981 GRE, all people taking exam:

\[ z = \frac{X - \mu}{\sigma} \]
\[ = \frac{600 - 489}{126} \]
\[ = .88 \quad p(\text{larger portion}) = .81 \]

A GRE score of 600 would correspond to the 81st percentile.

3.15 October 1981 GRE, all seniors and nonenrolled college graduates:

\[ z = \frac{X - \mu}{\sigma} \]
\[ = \frac{600 - 507}{118} \]
\[ = .79 \quad p = .785 \]
\[ 586.591 = X \]

For seniors and nonenrolled college graduates, a GRE score of 600 is at the 79th percentile, and a score of 587 would correspond to the 75th percentile.

3.17 GPA scores:

\[ N = 88 \quad \bar{X} = 2.46 \quad s = .86 \quad \text{[calculated from data set]} \]
\[ z = \frac{X - \bar{X}}{s} \]
\[ .6745 = \frac{X - 2.46}{.86} \]
\[ 3.04 = X \]

The 75th percentile for GPA is 3.04.

3.19 There is no meaningful discrimination to be made among those scoring below the mean, and therefore all people who score in that range are given a T score of 50.

3.21 Weight gain data

None of these is very close to normal, but the post intervention weight is closest.

3.23 Salaries for assistant professors (1999-2000)

I expect that you would do reasonably well if you treated these as normally distributed, especially if you calculated a trimmed mean and a Winsorized standard deviation. The extreme salaries probably come from people who have either stayed at the rank of Assistant Professor for many years, possibly because they don’t have the highest degree in their field, or those who have come to the university with considerable nonacademic experience.