Chapter 16 - Analyses of Variance and Covariance as General Linear Models

16.1 Eye fixations per line of text for poor, average, and good readers:

a. Design matrix, using only the first subject in each group:

$$X = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & -1 \end{bmatrix}$$

b. Computer exercise:

$R^2 = .608 \quad SS_{reg} = 57.7333 \quad SS_{residual} = 37.2000$

c. Analysis of variance:

$\bar{X}_1 = 8.2000 \quad \bar{X}_2 = 5.6 \quad \bar{X}_3 = 3.4 \quad \bar{X}_. = 5.733$

$n_1 = 5 \quad n_2 = 5 \quad n_3 = 5 \quad N = 15 \quad \Sigma X = 86 \quad \Sigma X^2 =$ 588

$$SS_{total} = \Sigma X^2 - \frac{(\Sigma X)^2}{N} = 588 - \frac{86^2}{15} = 94.933$$

$$SS_{group} = n\Sigma\left(\bar{X}_j - \bar{X}_.\right)^2 = 5[(8.2000 - 5.733)^2 + (5.6 - 5.733)^2 + (3.4 - 5.733)^2]$$

$$= 57.733$$

$$SS_{error} = SS_{total} - SS_{group} = 94.933 - 57.733 = 37.200$$

Source	*df*	SS	MS	*F*
Group	2	57.733	28.867	9.312*
Error	12	37.200	3.100	
Total	14	94.933		

$* p < .05 \quad [F_{.05(2,12)} = 3.89]$

16.3 Data from Exercise 16.1, modified to make unequal ns:

$R^2 = .624 \quad SS_{reg} = 79.0095 \quad SS_{residual} = 47.6571$

Analysis of variance:

$\bar{X}_1 = 8.2000 \quad \bar{X}_2 = 5.8571 \quad \bar{X}_3 = 3.3333 \quad \bar{X}. = 5.7968$

$n_1 = 5$ $n_2 = 7$ $n_3 = 9$ $N = 21$ $\Sigma X = 112$ $\Sigma X^2 = 724$

$$SS_{total} = \Sigma X^2 - \frac{(\Sigma X)^2}{N} = 724 - \frac{112^2}{21} = 126.6666$$

$$SS_{group} = \Sigma n_j\left(\bar{X}_j - \bar{X}_.\right)^2 = 5[\left(8.2000 - 5.7968\right)^2 + 7\left(5.8571 - 5.7968\right)^2 + 9\left(3.3333 - 5.7968\right)^2]$$

$$= 79.0095$$

$$SS_{error} = SS_{total} - SS_{group} = 126.6666 - 79.0095 = 47.6571$$

Source	*df*	SS	MS	*F*
Group	2	79.0095	39.5048	14.92*
Error	18	47.6571	2.6476	
Total	20	126.6666		

$* p < .05$ $[F_{.05(2,18)} = 3.55]$

16.5 Relationship between Gender, SES, and Locus of Control:

a. Analysis of Variance:

		SES			
		Low	Average	High	Mean
Gender	Male	12.25	14.25	17.25	14.583
	Female	8.25	12.25	16.25	12.250
	Mean	10.25	13.25	16.75	13.417

$\Sigma X = 644$ $\Sigma X^2 = 9418$ $n = 8$ $N = 48$

$$SS_{total} = \Sigma X^2 - \frac{(\Sigma X)^2}{N} = 9418 - \frac{644^2}{48} = 777.6667$$

$$SS_{gender} = sn\Sigma\left(\bar{X}_{i.} - \bar{X}_{..}\right)^2 = 3(8)[\left(14.583 - 13.417\right)^2 + \left(12.250 - 13.417\right)^2]$$

$$= 65.333$$

$$SS_{SES} = gn\Sigma\left(\bar{X}_{.j} - \bar{X}_{..}\right)^2 = 2(8)[\left(10.25 - 13.417\right)^2 + \left(13.25 - 13.417\right)^2 + \left(16.75 - 13.417\right)^2]$$

$$= 338.6667$$

$$SS_{cells} = n\Sigma\left(\bar{X}_{ij} - \bar{X}_{..}\right)^2 = 8[\left(12.25 - 13.417\right)^2 + ... + \left(16.25 - 13.417\right)^2] = 422.6667$$

$$SS_{GS} = SS_{cells} - SS_{gender} - SS_{SES} = 422.6667 - 65.3333 - 338.6667 = 18.6667$$

$$SS_{error} = SS_{total} - SS_{cells} = 777.6667 - 422.6667 = 355.0000$$

Source	*df*	SS	MS	*F*

Gender	1	65.333	65.333	7.730*
SES	2	338.667	169.333	20.034*
G x *S*	2	18.667	9.333	1.104
Error	42	355.000	8.452	
Total	47	777.667		

$*p < .05 \quad [F_{.05(1,42)} = 4.08;\ F_{.05(2,42)} = 3.23]$

b. ANOVA summary table constructed from sums of squares calculated from design matrix:

$$SS_G = SS_{reg(\alpha,\beta,\alpha\beta)} - SS_{reg(\beta,\alpha\beta)} = 422.6667 - 357.3333 = 65.333$$

$$SS_S = SS_{reg(\alpha,\beta,\alpha\beta)} - SS_{reg(\alpha,\alpha\beta)} = 422.6667 - 84.0000 = 338.667$$

$$SS_{GS} = SS_{reg(\alpha,\beta,\alpha\beta)} - SS_{reg(\alpha,\beta)} = 422.6667 - 404.000 = 18.667$$

$$SS_{total} = SS_Y = 777.667$$

The summary table is exactly the same as in part a (above).

16.7 The data from Exercise 16.5 modified to make unequal *n*s:

$$SS_{error} = SS_Y - SS_{reg(\alpha,\beta,\alpha\beta)} = 750.1951 - 458.7285 = 291.467$$

$$SS_G = SS_{reg(\alpha,\beta,\alpha\beta)} - SS_{reg(\beta,\alpha\beta)} = 458.7285 - 398.7135 = 60.015$$

$$SS_S = SS_{reg(\alpha,\beta,\alpha\beta)} - SS_{reg(\alpha,\alpha\beta)} = 458.7285 - 112.3392 = 346.389$$

$$SS_{GS} = SS_{reg(\alpha,\beta,\alpha\beta)} - SS_{reg(\alpha,\beta)} = 458.7285 - 437.6338 = 21.095$$

Source	*df*	SS	MS	*F*
Gender	1	60.015	60.015	7.21*
SES	2	346.389	173.195	20.80*
G x *S*	2	21.095	10.547	1.27
Error	35	291.467	8.328	
Total	40			

$*p < .05 \quad [F_{.05(1,35)} = 4.12;\ F_{.05(2,35)} = 3.27]$

16.9 Model from data in Exercise 16.5:

$$1.1667A_1 - 3.1667B_1 - 0.1667B_2 + 0.8333AB_{11} - 0.1667AB_{12} + 13.4167$$

Means: SES (*B*)

	Low	Avg	High

Gender (A)	Male	12.25	14.25	17.25	14.583
	Female	8.25	12.25	16.25	12.250
		10.25	13.25	16.75	13.417

$$\hat{\mu} = \overline{X}_{..} = 13.4167 = b_0 = \text{intercept}$$
$$\hat{\alpha}_1 = \overline{A}_1 - \overline{X}_{..} = 14.583 - 13.4167 = 1.1667 = b_1$$
$$\hat{\beta}_1 = \overline{B}_1 - \overline{X}_{..} = 10.25 - 13.4167 = -3.1667 = b_2$$
$$\hat{\beta}_2 = \overline{B}_2 - \overline{X}_{..} = 13.25 - 13.4167 = -0.1667 = b_3$$
$$\hat{\alpha\beta}_{11} = \overline{AB}_{11} - \overline{A}_1 - \overline{B}_1 + \overline{X}_{..} = 12.25 - 14.583 - 10.25 + 13.4167 = 0.8337 = b_4$$

$$\hat{\alpha\beta}_{12} = \overline{AB}_{12} - \overline{A}_1 - \overline{B}_2 + \overline{X}_{..} = 14.25 - 14.583 - 13.250 + 13.4167 = -0.1667 = b_5$$

16.11 Does Method III really deal with unweighted means?

Means:

	B_1	B_2	weighted	unweighted
A_1	4	10	8.5	7.0
A_2	10	4	8.0	7.0
weighted	8.0	8.5	8.29	
unweighted	7.0	7.0		7.0

The full model produced by Method 1: $\hat{Y} = 0.0A_1 + 0.0B_1 - 3.0AB_{11} + 7.0$

Effects calculated on weighted means:

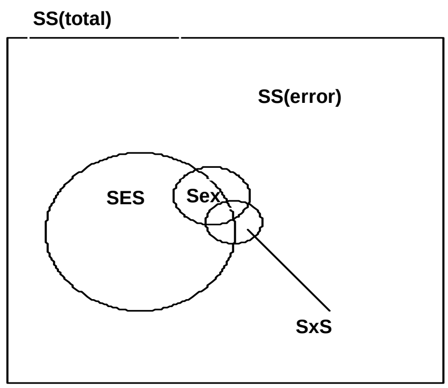
$$\hat{\mu} = \overline{X}_{..} = 8.29 \neq b_0 = \text{intercept}$$
$$\hat{\alpha}_1 = \overline{A}_1 - \overline{X}_{..} = 8.5 - 8.29 = .21 \neq b_1$$
$$\hat{\beta}_1 = \overline{B}_1 - \overline{X}_{..} = 8.0 - 8.29 = -.29 \neq b_2$$
$$\hat{\alpha\beta}_{11} = \overline{AB}_{11} - \overline{A}_1 - \overline{B}_1 + \overline{X}_{..} = 4.00 - 8.5 - 8.0 + 8.29 = -4.21 \neq b_3$$

Effects calculated on unweighted means:

$$\hat{\mu} = \overline{X}_{..} = 7.00 = b_0 = \text{intercept}$$
$$\hat{\alpha}_1 = \overline{A}_1 - \overline{X}_{..} = 7.0 - 7.0 = 0.0 = b_1$$
$$\hat{\beta}_1 = \overline{B}_1 - \overline{X}_{..} = 7.0 - 7.0 = 0.0 = b_2$$
$$\hat{\alpha\beta}_{11} = \overline{AB}_{11} - \overline{A}_1 - \overline{B}_1 + \overline{X}_{..} = 4.00 - 7.0 - 7.0 + 7.0 = -3.0 = b_3$$

These coefficients found by the model clearly reflect the effects computed on unweighted means. Alternately, carrying out the complete analysis leads to SS_A = SS_B = 0.00, again reflecting equality of unweighted means.

16.13 Venn diagram representing the sums of squares in Exercise 16.7:



16.15 Energy consumption of families:

a. Design matrix, using only the first entry in each group for illustration purposes:

$$X = \begin{bmatrix} 1 & 0 & 58 & 75 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 1 & 60 & 70 \\ \cdots & \cdots & \cdots & \cdots \\ -1 & -1 & 75 & 80 \end{bmatrix}$$

b. Analysis of covariance:

$$SS_{reg(\alpha,cov,\alpha c)} = 2424.6202$$

$$SS_{reg(\alpha,cov)} = 2369.2112$$

$$SS_{residual} = 246.5221 = SS_{error}$$

There is not a significant decrement in SS_{reg} and thus we can continue to assume homogeneity of regression.

$$SS_{reg(\alpha)} = 1118.5333$$

$$SS_{cov} = SS_{reg(\alpha,cov)} - SS_{reg(\alpha)} = 2369.2112 - 1118.5333 = 1250.6779$$

$$SS_{reg(cov)} = 1716.2884$$

$$SS_A = SS_{reg(\alpha,cov)} - SS_{reg(cov)} = 2369.2112 - 1716.2884 = 652.9228$$

Source	*df*	SS	MS	*F*
Covariate	1	1250.6779	1250.6779	55.81*
A (Group)	2	652.9228	326.4614	14.57*
Error	11	246.5221	22.4111	
Total	14	2615.7333		

$*p < .05$ $[F_{.05(1,11)} = 4.84;\ F_{.05(2,11)} = 3.98]$

16.17 Adjusted means for the data in Exercise 16.16:

(The order of the means may differ depending on how you code the group membership and how the software sets up its design matrix. But the numerical values should agree.)

$$\hat{Y} = -7.9099A_1 + 0.8786A_2 - 2.4022B_1 + 0.5667AB_{11} + 0.1311AB_{21} + 0.7260C + 6.3740$$

$$\hat{Y}_{11} = -7.9099(1) + 0.8786(0) - 2.4022(1) + 0.5667(1) + 0.1311(0) + 0.7260(61.3333) + 6.3740 = 41.1566$$

$$\hat{Y}_{12} = -7.9099(1) + 0.8786(0) - 2.4022(-1) + 0.5667(-1) + 0.1311(0) + 0.7260(61.3333) + 6.3740 = 44.8276$$

$$\hat{Y}_{21} = -7.9099(0) + 0.8786(1) - 2.4022(1) + 0.5667(0) + 0.1311(1) + 0.7260(61.3333) + 6.3740 = 49.5095$$

$$\hat{Y}_{22} = -7.9099(0) + 0.8786(1) - 2.4022(-1) + 0.5667(0) + 0.1311(-1) + 0.7260(61.3333) + 6.3740 = 54.0517$$

$$\hat{Y}_{31} = -7.9099(-1) + 0.8786(-1) - 2.4022(1) + 0.5667(-1) + 0.1311(-1) + 0.7260(61.3333) + 6.3740 = 54.8333$$

$$\hat{Y}_{32} = -7.9099(-1) + 0.8786(-1) - 2.4022(-1) + 0.5667(1) + 0.1311(1) + 0.7260(61.3333) + 6.3740 = 61.0333$$

(We enter 61.3333 for the covariate in each case, because we want to estimate what the cell means would be if the observations in those cells were always at the mean of the covariate.)

16.19 Klemchuk, Bond, & Howell (1990)

Tests of Between-Subjects Effects

Dependent Variable:DV

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	15.728[a]	3	5.243	8.966	.000
Intercept	2.456	1	2.456	4.201	.048
Daycare	2.640	1	2.640	4.515	.041
Age	11.703	1	11.703	20.016	.000
Daycare * Age	.037	1	.037	.064	.802
Error	21.050	36	.585		
Total	46.111	40			
Corrected Total	36.778	39			

a. R Squared = .428 (Adjusted R Squared = .380)

16.21 Analysis of GSIT in Mireault.dat:

Tests of Between-Subjects Effects

Dependent Variable: GSIT

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	1216.924[a]	5	243.385	2.923	.013
Intercept	1094707.516	1	1094707.516	13146.193	.000
GENDER	652.727	1	652.727	7.839	.005
GROUP	98.343	2	49.172	.590	.555
GENDER * GROUP	419.722	2	209.861	2.520	.082
Error	30727.305	369	83.272		
Total	1475553.000	375			
Corrected Total	31944.229	374			

a. R Squared = .038 (Adjusted R Squared = .025)

Estimated Marginal Means

GENDER * GROUP

Dependent Variable: GSIT

				95% Confidence Interval	
GENDER	GROUP	Mean	Std. Error	Lower Bound	Upper Bound
Male	1	62.367	1.304	59.804	64.931
	2	64.676	1.107	62.500	66.853
	3	63.826	1.903	60.084	67.568
Female	1	62.535	.984	60.600	64.470
	2	60.708	.858	59.020	62.396
	3	58.528	1.521	55.537	61.518

16.23 Analysis of variance on the covariate from Exercise 16.22.

The following is abbreviated SAS output.

```
                         General Linear Models Procedure

Dependent Variable: YEARCOLL
                                        Sum of          Mean
Source                    DF            Squares         Square       F Value
Pr > F
Model                      5            13.3477645      2.6695529      2.15
0.0600
Error                    292            363.0012288     1.2431549
Corrected Total          297            376.3489933

                   R-Square             C.V.              Root MSE          YEARCOLL Mean
                   0.035466             41.53258          1.11497               2.6845638

Source                    DF            Type III SS     Mean Square    F Value
Pr > F
GENDER                     1            5.95006299      5.95006299     4.79
0.0295
GROUP                      2            0.78070431      0.39035216     0.31
0.7308
GENDER*GROUP               2            2.96272310      1.48136155     1.19
0.3052

                GENDER         GROUP    YEARCOLL
                                        LSMEAN
                   1              1     2.27906977
                   1              2     2.53225806
                   1              3     2.68421053
                   2              1     2.88888889
                   2              2     2.85000000
                   2              3     2.70967742
```

These data reveal a significant difference between males and females in terms of YearColl. Females are slightly ahead of males. If the first year of college is in fact more stressful than later years, this could account for some of the difference we found in Exercise 16.21.

16.25 Everitt compared two therapy groups and a control group treatment for anorexia. The groups differed significantly in posttest weight when controlling for pretest weight ($F = 8.71$, $p < .0001$, with the Control group weighing the least at posttest. When we examine the difference between just the two treatment groups at posttest, the F does not reach significant, $F = 3.745$, $p = .060$, though the effect size for the difference between means (again controlling for pretest weights) is 0.62 with the Family Therapy group weighing about six pounds more than the Cognitive/Behavior Therapy group. It is difficult to know just how to interpret that result given the nonsignificant F.

16.27 A slope of 1.0 would mean that the treatment added a constant to people's pretest scores, which seems somewhat unlikely. Students might try taking any of the data in the book with a pretest and posttest score and plotting the relationship.

This relationship between difference scores and the analysis of covariance would suggest that in general an analysis of covariance might be the preferred approach. The only timemight think otherwise is when the difference score is really the measure of interest.