Chapter 14 – Repeated-Measures Designs

Note: As in previous chapters, there will be substantial rounding in these answers. I have attempted to make the answers fit with the correct values, rather than the exact results of the specific calculations shown here. Thus I may round cell means to two decimals, but calculation is carried out with many more decimals.

14.1 Does taking the GRE repeatedly lead to higher scores?

a. Statistical model:

$$X_{ij} = \mu + \pi_i + \tau_j + \pi\tau_{ij} + e_{ij} \text{ or } X_{ij} = \mu + \pi_i + \tau_j + e_{ij}'$$

b. Analysis:

Subject	Mean
1	566.67
2	450.00
3	616.67
4	663.33
5	436.67
6	696.67
7	503.33
8	573.33
Mean	563.33

Test Session	Mean
1	552.50
2	563.75
3	573.75

$$SS_{total} = \Sigma X^2 - \frac{(\Sigma X)^2}{N}$$

$$= 7811200 - \frac{(13520)^2}{24} = 194933.33$$

$$SS_{subj} = t\Sigma\left(\bar{X}_{i.} - \bar{X}_{..}\right)^2$$

$$= 3[\left(566.67 - 563.33\right)^2 + ... + \left(573.33 - 563.33\right)^2] = 3\left(63222.22\right) = 189,666.67$$

$$SS_{test} = n\Sigma\left(\bar{X}_{.j} - \bar{X}_{..}\right)^2 = 8[\left(552.50 - 563.33\right)^2 + (563.75 - 563.33)^2 + \left(573.75 - 563.33\right)^2]$$

$$= 8\left[226.04\right] = 1808.33$$

$$SS_{error} = SS_{total} - SS_{subj} - SS_{test}$$

$$= 194,933.33 - 189,666.67 - 1808.33 = 3458.33$$

Source	df	SS	MS	F
Subjects	7	189,666.66		
Within subj	16	5266.67		
Test session	2	1808.33	904.17	3.66 ns
Error	14	3458.33	247.02	
Total	23	194,933.33		

14.3 Teaching of self-care skills to severely retarded children:

Cell means:

		Phase		
		Baseline	Training	Mean
Group:	Exp	4.80	7.00	5.90
	Control	4.70	6.40	5.55
	Mean	4.75	6.70	5.72

Subject means:

		S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
Grp	Exp	8.5	6.0	2.5	6.0	5.5	6.5	6.5	5.5	5.5	6.5
	Control	4.0	5.0	9.0	3.5	4.0	8.0	7.5	4.5	5.0	5.5

$\Sigma X^2 = 1501$ $\quad \Sigma X = 229$ $\quad N = 40$ $\quad n = 10$ $\quad g = 2$ $\quad p = 2$

$$SS_{\text{total}} = \Sigma X^2 - \frac{(\Sigma X)^2}{N} = 1501 - \frac{(229)^2}{40} = 189.975$$

$$SS_{subj} = p\Sigma\left(\bar{X}_{ij.} - \bar{X}_{...}\right)^2$$

$$= 2[\left(8.5 - 5.72\right)^2 + \ldots + (5.5 - 5.72)^2] = 106.475$$

$$SS_{group} = pn\Sigma\left(\bar{X}_{..k} - \bar{X}_{...}\right)^2$$

$$= 2\left(8\right)[\left(5.90 - 5.72\right)^2 + \left(5.55 - 5.72\right)^2] = 1.225$$

$$SS_{phase} = gn\Sigma\left(\bar{X}_{.j.} - \bar{X}_{...}\right)^2$$

$$= 2\left(10\right)[\left(4.75 - 5.72\right)^2 + \left(6.70 - 5.72\right)^2] = 38.025$$

$$SS_{cells} = n\Sigma\left(\bar{X}_{.jk} - \bar{X}_{...}\right)^2$$

$$= 10\left[\left(4.80 - 5.72\right)^2 + \ldots + \left(6.40 - 5.72\right)^2\right] = 39.875$$

$$SS_{PG} = SS_{cells} - SS_{phase} - SS_{group} = 39.875 - 38.025 - 1.225 = 0.925$$

Source	*df*	SS	MS	*F*
Between Subj	19	106.475		
Groups	1	1.125	1.125	0.19
Ss w/in Grps	18	105.250	5.847	
Within Subj	20	83.500		
Phase	1	38.025	38.025	15.26*
P x G	1	0.625	0.625	0.25
P x Ss w/in Grps	18	44.850	2.492	
Total	39	189.975		

$* p < .05 \quad [F_{.05(1,18)} = 4.41]$

There is a significant difference between baseline and training, but there are no group differences nor a group x phase interaction.

14.5 Adding a No Attention control group to the study in Exercise 14.3:

Cell means:

		Phase		
		Baseline	Training	Total
Group	Exp	4.8	7.0	5.90
	Att Cont	4.7	6.4	5.55
	No Att Cont	5.1	4.6	4.85
	Total	4.87	6.00	5.43

Subject means:

Group:	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
Exp	8.5	6.0	2.5	6.0	5.5	6.5	6.5	5.5	5.5	6.5
Att Cont	4.0	5.0	9.0	3.5	4.0	8.0	7.5	4.5	5.0	5.0
No Att Cont	3.5	5.0	7.0	5.5	4.5	6.5	6.5	4.5	2.5	3.0

$\Sigma X^2 = 2026 \qquad \Sigma X = 326 \qquad N = 60 \qquad n = 10 \qquad g = 3 \qquad p = 2$

$$SS_{Total} = \Sigma X^2 - \frac{(\Sigma X)^2}{N} = 2026 - \frac{(326)^2}{40} = 254.7333$$

$$SS_{subj} = p\Sigma\left(\bar{X}_{ij.} - \bar{X}_{...}\right)^2$$
$$= 2[\left(8.5 - 5.43\right)^2 + ... + (3.0 - 5.43)^2] = 159.733$$

$$SS_{group} = pn\Sigma\left(\bar{X}_{..k} - \bar{X}_{...}\right)^2$$
$$= 2\left(8\right)[\left(5.90 - 5.43\right)^2 + \left(5.55 - 5.43\right)^2 + \left(4.85 - 5.43\right)^2] = 11.433$$

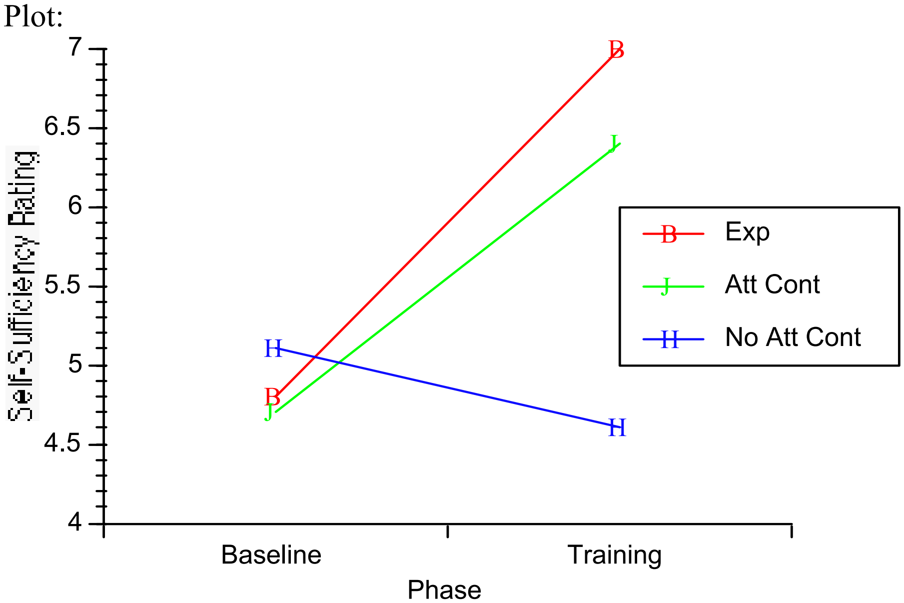
$$SS_{phase} = gn\Sigma\left(\bar{X}_{.j.} - \bar{X}_{...}\right)^2$$
$$= 3(10)[(4.87 - 5.43)^2 + (6.00 - 5.43)^2] = 19.267$$

$$SS_{cells} = n\Sigma\left(\bar{X}_{.jk} - \bar{X}_{...}\right)^2$$
$$= 10\left[(4.80 - 5.43)^2 + \ldots + (4.60 - 5.43)^2\right] = 52.333$$
$$SS_{PG} = SS_{cells} - SS_{phase} - SS_{group} = 51.333 - 19.267 - 11.433 = 20.633$$

Source	*df*	SS	MS	*F*
Between subj	29	159.7333		
Groups	2	11.4333	5.7166	1.04
Ss w/ Grps	27	148.300	5.4926	
Within subj	30	95.0000		
Phase	1	19.2667	19.2667	9.44*
P * G	2	20.6333	10.3165	5.06*
P * Ss w/Grps	27	55.1000	2.0407	
Total	59	254.733		

$*p < .05 \quad [F_{.05(1,27)} = 4.22;\ F_{.05(2,27)} = 3.36]$

b. Plot:



c. There seems to be no difference between the Experimental and Attention groups, but both show significantly more improvement than the No Attention group.

14.7 For the data in Exercise 14.6:

a. Variance-covariance matrices:

$$\hat{\Sigma}_{owners} = \begin{bmatrix} 1.30 & 1.50 & 0.75 \\ 1.50 & 2.00 & 1.00 \\ 0.75 & 1.00 & 1.00 \end{bmatrix}$$

$$\hat{\Sigma}_{non-owners} = \begin{bmatrix} 2.70 & 1.20 & 1.85 \\ 1.20 & 0.70 & 0.60 \\ 1.85 & 0.60 & 3.30 \end{bmatrix}$$

$$\hat{\Sigma}_{pooled} = \begin{bmatrix} 2.00 & 1.35 & 1.30 \\ 1.35 & 1.35 & 0.80 \\ 1.30 & 0.80 & 2.15 \end{bmatrix} \quad \begin{matrix} \bar{s}_j \\ 1.550 \\ 1.167 \\ 1.417 \end{matrix}$$

$$\hat{\Sigma}_{between} = \begin{bmatrix} 0.18 & 0.36 & 1.38 \\ 0.36 & 0.72 & 2.76 \\ 1.38 & 2.76 & 10.58 \end{bmatrix}$$

b. $\hat{e}$

$$\bar{s}_{jj} = \frac{2.00 + 1.35 + 2.15}{3} = 1.833$$

$$\bar{s} = \frac{2.00 + ... + 2.15}{9} = 1.378$$

$$\Sigma s_{jk}^2 = 2.00^2 + \ldots + 2.15^2 = 18.750$$

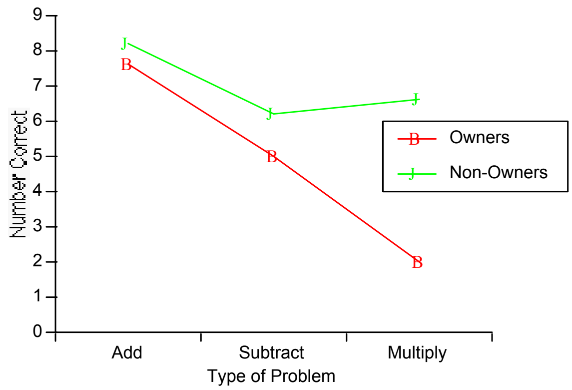
$$\Sigma \bar{s}_j^2 = 1.550^2 + 1.167^2 1.417^2 = 18.750$$

$$\hat{e} = \frac{b^2\left(\bar{s}_{jj} - \bar{s}\right)^2}{(b-1)\left(\Sigma s_{jk}^2 - 2b\Sigma\bar{s}_j^2 + b^2\bar{s}^2\right)} = \frac{9(1.833 - 1.378)^2}{2\left[18.75 - 6(5.772) + 9\left(1.378^2\right)\right]}$$

$$= \frac{1.863}{2.406} = 0.771$$

14.9 Back to the calculator usage data in Exercise 14.6:

a. Plot



b. Analysis of simple effects:

Cell means:

	Problems			
	Add	Subt	Mult	Mean
Owners	7.6	5.0	2.0	4.87
Non-Owners	8.2	6.2	6.6	7.00
Mean	7.9	5.6	4.3	5.935

For the first three simple effects we're breaking down a combination of SS_{Groups} and SS_{PG} from the overall analysis in Exercise 14.6, so for the error term we will need to combine the error terms which were used to test MS_{Groups} and MS_{PG} in that analysis.

$$MS_{w/in Cell} = \frac{MS_{Ss\ w/in\ grps} + MS_{P*Ss\ w/in\ grps}}{df_{Ss\ w/in\ grps} + df_{P*Ss\ w/in\ grps}} = \frac{33.066 + 10.934}{8 + 16} = 1.833$$

The *df* against which to evaluate *F* also must be adjusted for these simple effects. F_{obt} will be evaluated against $F_{.05(g-1, f')}$ but we must first calculate f'.

$$f' = \frac{(u+v)^2}{\frac{u^2}{df_u} + \frac{v^2}{df_v}} = \frac{(33.066 + 10.934)^2}{\frac{33.066^2}{8} + \frac{10.934^2}{16}} = 13.431$$

$u = SS_{Ss\ w/in\ grps}$

$v = SS_{P*Ss\ w/in\ grps}$

With the error terms and degrees of freedom ready, we go ahead with calculating the sums of squares and testing them:

$$SS_{group\ \text{at}\ Add} = n\Sigma\left(\bar{X}_{i1} - \bar{X}_{.1}\right)^2$$
$$= 5[\left(7.6 - 7.9\right)^2 + \left(8.2 - 7.9\right)^2] = 0.9$$
$$MS_{group\ \text{at}\ Add} = \frac{SS_{group\ \text{at}\ Add}}{df_{group\ \text{at}\ Add}} = \frac{0.9}{1} = 0.9$$
$$F_{group\ \text{at}\ Add} = \frac{MS_{group\ \text{at}\ Add}}{MS_{w/in\ cells}} = \frac{0.9}{1.833} = < 1$$

$$SS_{group\ at\ Subt} = n\Sigma\left(\bar{X}_{i2} - \bar{X}_{.2}\right)^2$$
$$= 5[\left(5.0 - 5.6\right)^2 + \left(6.0 - 5.6\right)^2] = 3.6$$
$$MS_{group\ at\ Subt} = \frac{SS_{group\ at\ Subt}}{df_{group\ at\ Subt}} = \frac{3.6}{1} = 3.6$$
$$F_{group\ at\ Subt} = \frac{MS_{group\ at\ Subt}}{MS_{w/in\ cells}} = \frac{3.6}{1.833} = 1.96ns$$

$$SS_{group\ at\ Mult} = n\Sigma\left(\bar{X}_{i3} - \bar{X}_{.3}\right)^2$$
$$= 5[\left(2.0 - 4.3\right)^2 + \left(6.6 - 4.3\right)^2] = 52.9$$
$$MS_{group\ at\ Mult} = \frac{SS_{group\ at\ Mult}}{df_{group\ at\ Mult}} = \frac{52.9}{1} = 52.9$$
$$F_{group\ at\ Mult} = \frac{MS_{group\ at\ Mult}}{MS_{w/in\ cells}} = \frac{52.9}{1.833} = 28.86\,*$$

$*\,p < .05 \quad [F_{.05(g-1,f')} = F_{.05(1,13)} = 4.67]$

$SS_{\text{prob at calc}} = 78.53$ $F = 112.19$*

$SS_{\text{prob at noncalc}} = 11.20$ $F = 5.51$*

For the last two simple effects we're breaking down a combination of SS_{Problems} and SS_{PG} from the overall analysis in Exercise 14.6. Since MS_{Problems} and MS_{PG} were both tested by the same error term in that analysis ($MS_{\text{P*Ss w/in grps}}$) we could use that error term to test these simple effects. However, as pointed out in the chapter, violations of sphericity create serious problems when testing simple effects, and for that reason we will use separate error terms for the two analyses. The easiest way to do this is to run separate repeated measures analysis of variance for each group. This will produce the same sums of squares for the simple effect, as well as the appropriate error term.

The following results were produced by SPSS. Notice that the form of the printout is quite different from what we usually have. The corrected *df* (assuming a lack of sphericity) are given, as is the resulting significance level. (SPSS adjusts the two mean squares as the *df* are adjusted, but that does not alter the resulting *F*.)

Calculator owners:

Tests of Within-Subjects Effects[a]

Measure: MEASURE_1

Source		Type III Sum of Squares	df	Mean Square	F	Sig.
PROBLEM	Sphericity Assumed	78.533	2	39.267	112.190	.000
	Greenhouse-Geisser	78.533	1.477	53.157	112.190	.000
	Huynh-Feldt	78.533	2.000	39.267	112.190	.000
	Lower-bound	78.533	1.000	78.533	112.190	.000
Error(PROBLEM)	Sphericity Assumed	2.800	8	.350		
	Greenhouse-Geisser	2.800	5.910	.474		
	Huynh-Feldt	2.800	8.000	.350		
	Lower-bound	2.800	4.000	.700		

a. Calculator Owner = Owner

Non-owners:

Tests of Within-Subjects Effects[a]

Measure: MEASURE_1

Source		Type III Sum of Squares	df	Mean Square	F	Sig.
PROBLEM	Sphericity Assumed	11.200	2	5.600	5.508	.031
	Greenhouse-Geisser	11.200	1.564	7.159	5.508	.047
	Huynh-Feldt	11.200	2.000	5.600	5.508	.031
	Lower-bound	11.200	1.000	11.200	5.508	.079
Error(PROBLEM)	Sphericity Assumed	8.133	8	1.017		
	Greenhouse-Geisser	8.133	6.258	1.300		
	Huynh-Feldt	8.133	8.000	1.017		
	Lower-bound	8.133	4.000	2.033		

a. Calculator Owner = NonOwner

The *F* is significant in both cases, indicating that Task made a difference. If we had used a pooled error term, MS_{error} would have been 0.683, which, because of the fact that the sample sizes were equal, is the average of the two error terms we used. But notice that the pooled error term would have been about 60% of what it was when we treat the groups separately.

14.11 From Exercise 14.10:

a. Simple effect of reading ability for children:

$$SS_{R\,at\,C} = in\Sigma\left(\bar{X}_{R\,at\,C} - \bar{X}_C\right)^2$$

$$= 3(5)[(4.80 - 3.50)^2 + (2.20 - 3.50)^2] = 50.70$$

$$MS_{R\,at\,C} = \frac{SS_{R\,at\,C}}{df_{R\,at\,C}} = \frac{50.70}{1} = 50.70$$

Because we are using only the data from Children, it would be wise not to use a pooled error term. The following is the relevant printout from SPSS for the Between-subject effect of Reader.

Tests of Between-Subjects Effects[a]

Measure: MEASURE_1

Transformed Variable: Average

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Intercept	367.500	1	367.500	84.483	.000
READERS	50.700	1	50.700	11.655	.009
Error	34.800	8	4.350		

a. AGE = Children

b. Simple effect of items for adult good readers:

$$SS_{I\,at\,AG} = n\Sigma\left(\bar{X}_{I\,at\,AG} - \bar{X}_{AG}\right)^2$$

$$= 5[(6.20 - 5.73)^2 + (6.00 - 5.73)^2 + (5.00 - 5.73)^2] = 4.133$$

Again, we do not want to pool error terms. The following is the relevant printout from SPSS for Adult Good readers. The difference is not significant, nor would it be for any decrease in the *df* if we used a correction factor.

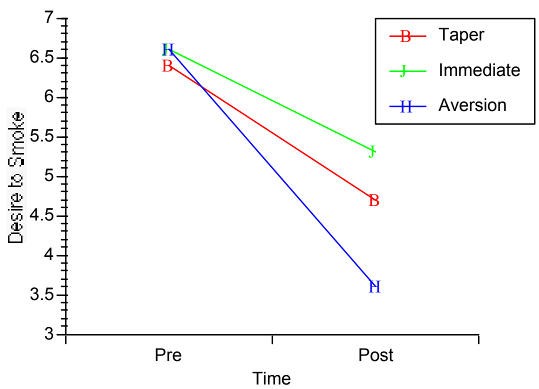
Tests of Within-Subjects Effects

Measure: MEASURE_1
Sphericity Assumed

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
ITEMS	4.133	2	2.067	3.647	.075
Error(ITEMS)	4.533	8	.567		

14.13 It would certainly affect the covariances because we would force a high level of covariance among items. As the number of responses classified at one level of Item went up, another item would have to go down.

14.15 Plot of results in Exercise 14.14:



14.17 Analysis of data in Exercise 14.5 by BMDP:

a. Comparison with results obtained by hand in Exercise 14.5.

b. The F for Mean is a test on H_0: $\mu = 0$.

c. $MS_{w/in}$ Cell is the average of the cell variances.

14.19 Source column of summary table for 4-way ANOVA with repeated measures on A & B and independent measures on C & D.

Source
Between *S*s
C
D
CD
*S*s w/in groups
Within Ss
A
AC
AD
ACD
A x *Ss* w/in groups
B
BC
BD
BCD
B x *S*s w/in groups
AB
ABC
ABD
ABCD
AB x *S*s w/in groups
Total

14.21 Using Manova in Exercise 14.20 we have gained freedom from the sphericity assumption, but at the potential loss of a small amount of power.

14.23 Analysis of Stress data:

Source	*df*	SS	MS	*F*	Pillai *F*	Prob
Between subj	97	137.683				
Gender	1	7.296	7.296	5.64*		
Role	1	8.402	8.402	6.49*		
G * R	1	0.298	0.298	<1		
Ss w/in Grps	94	121.687	1.294			
Within subj	97	87.390				
Time	1	1.064	1.064	1.23*	1.23	0.2700
T*G	1	0.451	0.451	<1	0.52	0.4720
T*R	1	0.001	0.001	<1	0.00	0.9708
T*G*R	1	4.652	4.652	5.38*	5.38	0.0225
T*Ss w/in grps	94	81.222	0.864			
Total	194	103.386				

*$p < .05$

The univariate and multivariate F values agree because we have only two levels of each independent variable.

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