MU 209 Handout 2
(based largely on material of Dmitri Tymoczko, Princeton University)

More on scales

III. Deriving the diatonic scale. Historically, the diatonic scale came *before* the chromatic scale. Where did the diatonic scale come from? The basic answer is that the diatonic scale is maximally saturated with consonant intervals (such as the perfect fifth, major third, and minor third) while also dividing the octave nearly evenly and containing a reasonably small number of notes.

Suppose we want to invent a scale that is (1) nearly even; (2) has only 5-8 notes; and (3) contains as many consonant intervals as possible. Since the perfect fifth is the most consonant interval[1], we might try building a scale so that every note has a perfect fifth above it: so C is in the scale, then we would include G, and D above that, and so on. Unfortunately, a stack of acoustically perfect fifths will *never* return to its starting point. (Remember that the acoustically pure fifth is slightly different from the equal tempered fifth of the ordinary piano.) This means we need to cheat: one strategy is to use a stack of fifths C-G-D-A-E-… where the last note is *almost* (but note quite) a fifth away from the first note. There turn out to be three salient possibilities: the five-note *pentatonic* scale C-G-D-A-E, whose last note is 8 semitones below the first; the seven note *diatonic scale* F-C-G-D-A-E-B- whose last note is 6 semitones below the first; and the twelve-note *chromatic scale,* B♭-F-C-G-D-A-E-B-F♯-C♯-G♯-D♯ whose "wolf fifth" D♯-B♭ is about ¼ of a semitone smaller than the others. Contemporary equal temperament cheats in another way, altering the fifths in the chromatic scale so that they are all exactly equal.

These three scales all divide the octave nearly evenly while being maximally saturated with perfect fifths. However, the diatonic scale has two advantages: it has fewer notes than the chromatic, while containing more thirds than the pentatonic. In fact, every note in the diatonic scale has either a major or minor third above it.

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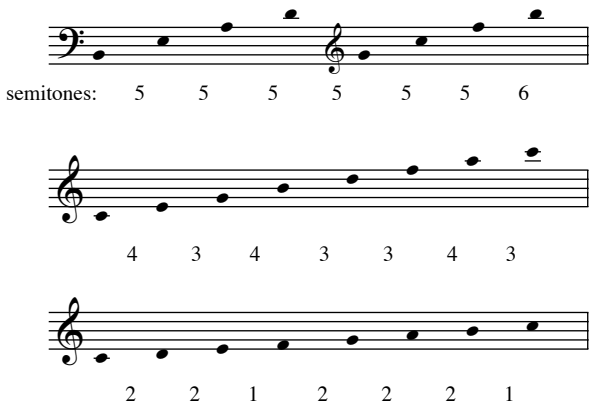
[1] Actually the octave is even more consonant. The requirement that your scale is maximally stuffed with octaves is equivalent to the requirement that every note in the scale have an octave above and below it—in other words, that the scale be *octave repeating,* having the same pattern in every octave. These scales are both easy to remember, and maximally saturated with the most consonant interval.

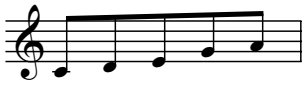


Here, we see that if we take every third note of the diatonic scale, we get a stack of perfect fourths, with only one augmented fourth. If we take every other note of the scale we get a stack of major and minor thirds.

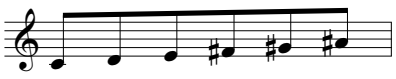
Note that if we tune the fifths as perfectly as possible, we get a slightly different diatonic scale than if we tune the thirds as perfectly as possible. The first diatonic scale is called the "Pythagorean" scale and has as many 3/2 (perfect fifth) ratios as possible; the second is called the "just" diatonic scale and has as many 5/4 (major third) and 6/5 (minor third) ratios as possible. They're not quite the same!

V. Deriving other familiar scales. There are a number of other scales that are important in Western (and other) music. All of these are saturated with consonant intervals, while also dividing the octave reasonably evenly.

For example, the *pentatonic scale* contains as many perfect fifths as a five-note scale can have (which is to say, four), while also dividing the octave reasonably evenly. The pentatonic scale has just two sizes of step (2 and 3 semitones large) and just two sizes of third (4 and 5 semitones large). It is enormously important, and was once—prior to the spread of Western music—the world's most popular scale.

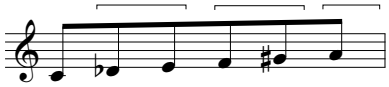
 **C pentatonic**

The *whole-tone scale* divides the octave into six precisely even parts and contains as many major thirds as a three-note scale can have. (That is, every note in the scale has a major third above it.) It was first used by Glinka, and was made famous by Debussy. It frequently appears in jazz (especially Thelonious Monk's playing) and is often used to signify dream sequences in television and movies.

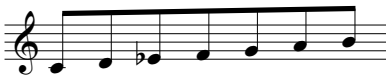
 **C whole tone**

If you transpose the scale by two chromatic semitones, you end up with the same scale again—meaning that there are just two whole-tone scales! Since this scale has very little internal variety, it quickly becomes boring.

A second six-note scale is the *hexatonic scale*; consisting of alternating one-semitone and three-semitone steps. It has as many major thirds as a scale can have (one above each note!) while also having three different perfect fifths and containing many different major and minor triads (how many, if you ignore spelling?)

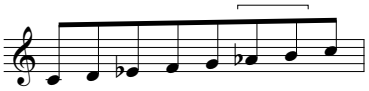

C hexatonic

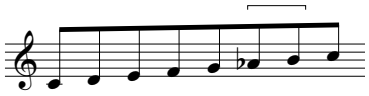
Among the important seven-note scales, we have the *ascending melodic minor* or *acoustic scale* (also sometimes called the "lydian dominant"), with major or minor thirds above each of its notes, and which divides the octave fairly evenly. It gets this name because it is the best approximation to the first 7 pitch classes in the overtone series.


C melodic minor or F acoustic

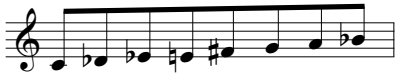
The ascending melodic minor has steps of 1 and 2 semitones and thirds of 3 and 4 semitones, just like the diatonic scale. The fourths, however, come in *three* sizes: 4 semitones (B-Ef), 5 semitones (D-G), and 6 semitones (Ef-A).

There are two other important seven-note scales, the harmonic minor and harmonic major scales. These scales have *three* step sizes: each scale has a single three-semitone step, as shown on the bracket. However, the scales are maximally saturated with major and minor thirds, just like the diatonic scale.


C harmonic minor


C harmonic major

Finally, there is the *octatonic* scale, which is maximally saturated with minor thirds, while also containing many perfect fifths and triads (how many?).


C octatonic or C diminished (jazz)

The octatonic scale has two sizes of step (1 and 2 semitones) and fourth (4 and 5 semitones). However, it has only one size of third (3 semitones) and fifth (6 semitones). In this sense it is nearly as symmetrical as the whole-tone scale: if we transpose the octatonic scale by three semitones, then we end up with the same scale again.

The following table summarizes our scales.

# of notes	Very even	Somewhat Less Even	Maximally Saturated with
Five notes	Pentatonic		perfect fifths
Six notes	Whole tone	Hexatonic	major thirds
Seven notes	Diatonic		perfect fifths
		Melodic minor (i.e. ascending)	thirds
		Harmonic minor	thirds
		Harmonic major	thirds
Eight notes	Octatonic		minor thirds

There are many others! An interesting book is Nicholas Slonimsky's *Thesaurus of Scales and Melodic Patterns* (243 oversized pages), which John Coltrane is said to have practiced from. But you can also just mess around and come up with your own. Slonimsky's scales are all constructed from the "menu" of the 12-tone chromatic scale. Still more possibilities emerge with alternate tuning systems, which include not just alternative temperaments of the familiar diatonic or chromatic scales, but also microtonal systems with more than 12 tones to the octave.

VI. A brief history of scales in Western music.

a) Medieval and Renaissance music (1100–1600) uses only the diatonic scale. Cadences on various scale degrees suggest all the modes except Locrian. Accidentals are rare, so the music typically employs the "white note" diatonic collection (C major/A natural minor). Unsystematic use of sharps and flats occasionally gives rise to other diatonic scales however.[2]

b) Classical tonality (1600–1875) limits the number of modes to two: Ionian or major, and aeolian or "natural minor." The desire for a "leading tone" (the note a semitone below the tonic) leads to three separate forms of the minor scale. Minor-key music switches freely (and rapidly) between these three forms.

c) The modern period (1875-now). In the late nineteenth century, composers became newly interested in the diatonic modes; this reawakening continued in the twentieth century. Since there were now 12 major scales, and each had 7 modes, we now had $12 \times 7 = 84$ different modes!

Composers also became interested in all the nondiatonic scales shown on this handout, as well as all of their modes. Musicians such as Debussy and Ravel, modern

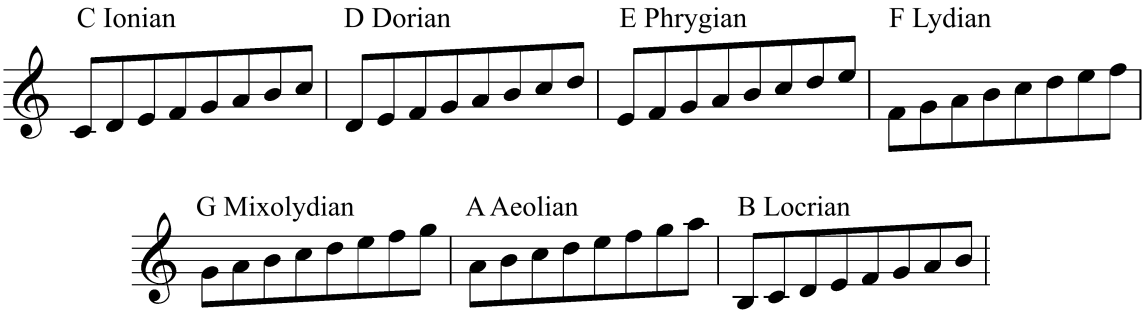
[2] This is a bit of an approximation. We know that performers often added extra accidentals when performing this music; but we don't know how often they did so. It's possible that the music we think is purely diatonic was actually performed in a way that we would consider very chromatic!

jazz improvisers, and minimalists such as Steve Reich, have developed an impressive facility with all the modes of all the different scales discussed here.

VII. Modes

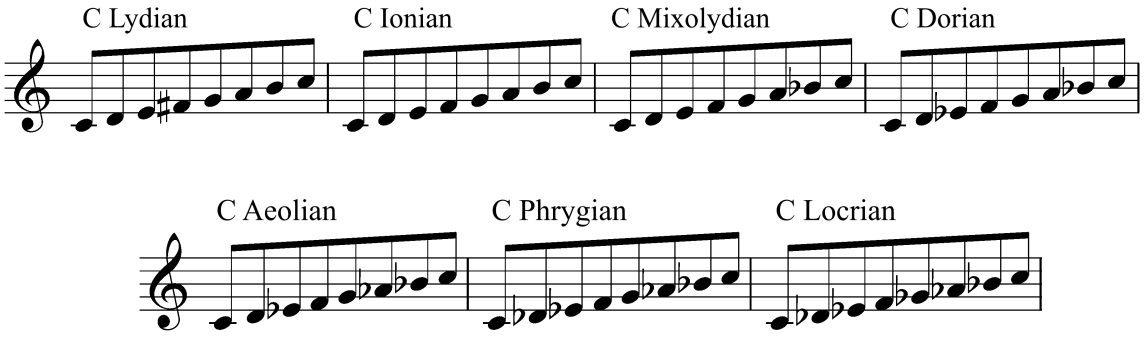
Because of the *asymmetry* of the diatonic and melodic minor scales (their large and small steps are not distributed exactly evenly), each has 12 distinct transpositions and a distinct mode on every scale step. The same is true of the familiar pentatonic scale: its 2-2-3-2-3 pattern is asymmetrical, so it yields 12 transpositions and 5 distinct mode types. By contrast, scales that have some degree of *symmetry* will have fewer than 12 distinct transpositions, and not every scale step will have its own unique modal type.

Here are the seven modes of the diatonic scale, along with their traditional names, which are derived from various ancient Greek tribes (!) for no particular reason. All were used in traditional musical practice except Locrian, which was a theoretical "leftover" that completes the system and was not used until the 20th century.

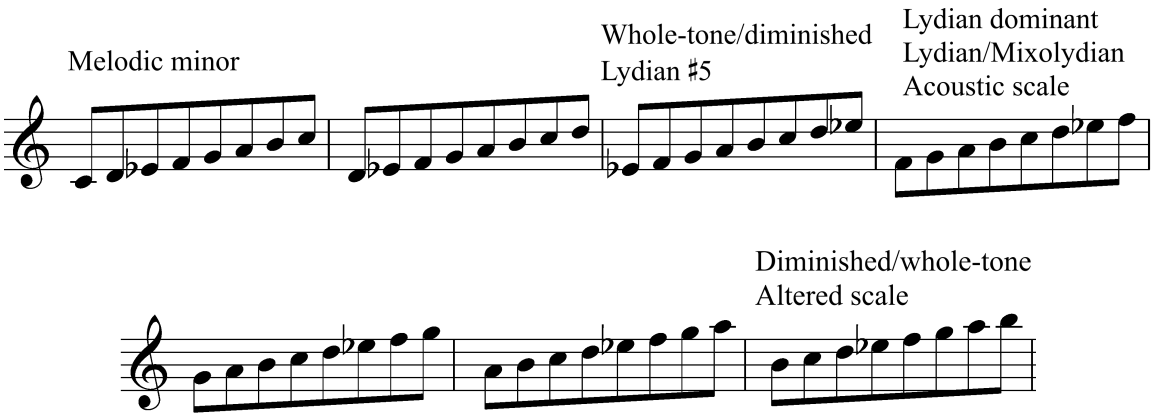


Note that a particular transposition is labeled by its *modal tonic*, not by the major scale it is "relative" to. For instance, E-F♯-G-A-B-C♯-D is E Dorian, not D Dorian, though it uses the notes of the D diatonic scale. (Popular musicians sometimes talk about transposition the other way, though, based on the major scale containing the same notes.)

Another was to think about the seven diatonic modes is in order from sharpest/brightest to flattest/darkest:

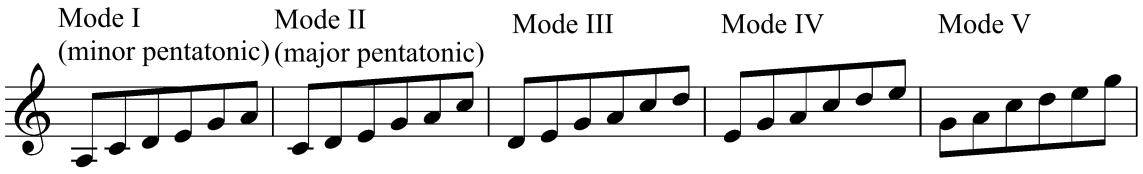


Here are all the modes of the melodic minor scale. They do not all have universally recognized names, and some have multiple common identifiers.

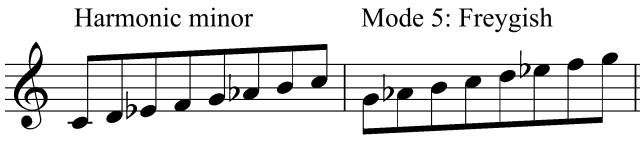


"Altered" is an abbreviation of "altered dominant" because it goes well with the so-called dom$^{7(alt)}$ chord in jazz (a dominant with ♭9, ♯9, ♭5, and ♭13) with the chord root as tonic of the mode. Jazz musicians learn which modes correspond to which chords: a full 13th chord (root, third, fifth, seventh, ninth, eleventh, thirteenth) is simply a 7-note scale stacked in thirds instead of steps!

The standard pentatonic scale (there are of course other possible 5-note scales) similarly has 5 distinct modes. Except for the major and minor (which are by far the most familiar, especially in Western music) there are not conventional names for them; theorists number them instead.



Also less common are most of the modes of the harmonic minor scale—but that makes them ripe for exploration! One is common in Klezmer music, where it is called Freygish (Yiddish for "Phrygian").



You should also familiarize yourself with the symmetrical scales discussed. Thankfully, they have fewer modes and transpositions to master.