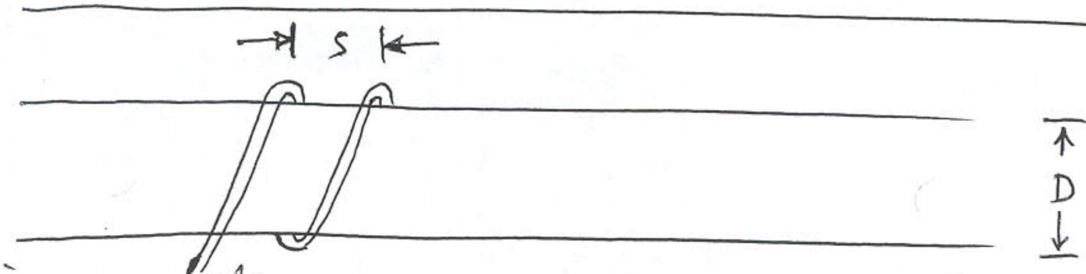
(1)

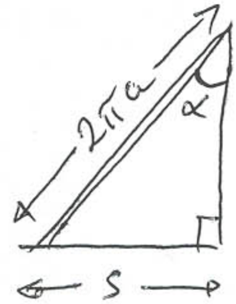
Clougherty
11/7/05

Axial wave number in helical resonator



Helix geometry

D = diameter of helix $= 2a$
s = pitch of helix
α = pitch angle



Oscillator

f = oscillator frequency
$k_0 \equiv \frac{2\pi f}{c}$ free space wavenumber

Treating helix as anisotropic conductor with perfect conductivity along windings and zero conductivity perpendicular to windings, the lowest mode must satisfy the following transcendental eqn

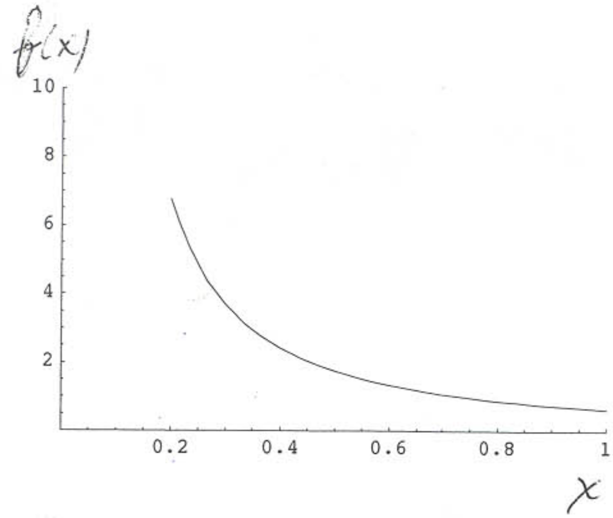
$$\frac{K_1(qa)\, I_1(qa)}{K_0(qa)\, I_0(qa)} = \frac{(qa)^2 \tan^2\alpha}{(k_0 a)^2}$$

where $q^2 = k_z^2 - k_0^2$ $\begin{cases} q = \text{radial wavenumber} \\ k_z = \text{axial wavenumber} \end{cases}$

Define function $f(x)$

$$f(x) \equiv \frac{K_1(x)\, I_1(x)}{K_0(x)\, I_0(x)} \frac{1}{x^2}$$

(2)



By iteration, we can solve transcendental eqn:

$$f(x) = \frac{\tan^2 \alpha}{(h_o a)^2}$$

<u>Measurements:</u>

$D = 0.9$ cm
n = turn density = 12 turns/cm
$f = 94$ MHz

Extract pitch from turn density

$$s = \frac{1}{n} = \frac{1}{12} \text{ cm}$$

Calculation of α: $\sin\alpha = \frac{s}{2\pi a} = \left(\frac{1}{12}\right)\frac{1}{\pi(.9)}$

$$\alpha \cong 2.9 \times 10^{-2} \text{ radians}$$

Calculation of h_o:

$$h_o = \frac{2\pi(94 \times 10^6)}{3 \times 10^{10}} \text{ cm}^{-1}$$

$$h_o = 1.97 \times 10^{-2} \text{ cm}^{-1} \simeq \underline{2 \times 10^{-2} \text{ cm}^{-1}}$$

③

$\therefore k_0 a = 8.9 \times 10^{-3} \; (\ll 1)$

Find x : $\quad f(x) = \dfrac{(2.9 \times 10^{-2})^2}{(8.9 \times 10^{-3})^2} \doteq 10.6$

$$\underline{x \simeq 0.15} \; (= q a)$$

$$(k_z a)^2 = (q a)^2 + (k_0 a)^2$$
$$= (0.15)^2 + (8.9 \times 10^{-3})^2$$
$$= (2.25 \times 10^{-2}) + (7.9 \times 10^{-5}) \simeq 2.25 \times 10^{-2}$$

$\therefore k_z \simeq q = \dfrac{0.15}{0.45}\ \mathrm{cm}^{-1} = \underline{0.3\ \mathrm{cm}^{-1}}$

$$\boxed{\lambda = \frac{2\pi}{k_z} = 19\ \mathrm{cm}}$$

Measured $\lambda = 21$ cm (9.5% error)