

Math 255 - Spring 2022  
Proofs and primes  
10 points

This contains proofs on the factorization of integers into primes. This homework is worth 10 points.

1. Show that if  $n > 4$  is a composite number, then  $n$  divides  $(n - 1)!$ .
2. Let  $n$  be composite, and let  $p$  be the smallest prime factor of  $n$ . Prove that if  $p > n^{1/3}$ , then  $\frac{n}{p}$  is a prime number.
3. True or false: Let  $n$  be a positive integer. If  $p$  and  $q$  are distinct primes that divide  $n$ , and each is greater than  $n^{1/4}$ , then  $\frac{n}{pq}$  is an integer and is prime.

If this statement is true, prove it. If it is false, give a counter-example.