

Math 255 - Spring 2022
Euler's ϕ -function
15 points

This homework invites you to prove some facts about the Euler ϕ -function. Please remember that you can always solve as many or as few problems as you would like, except if you are taking this course for graduate credit in which case you must attempt all problems of this homework.

1. Prove the following:

(a) If n is odd, then $\phi(2n) = \phi(n)$.

(b) If n is even, then $\phi(2n) = 2\phi(n)$.

(c) $\phi(n) = \frac{n}{2}$ if and only if $n = 2^k$ for some integer $k \geq 1$.

2. Prove that if n has r distinct prime factors, then $\phi(n) > \frac{n}{2^r}$.

3. Prove that for any two positive integers m and n with $\gcd(m, n) = d$, we have

$$\phi(n)\phi(m) = \phi(mn)\frac{\phi(d)}{d}.$$