Math 255 - Spring 2022
Another multiplicative function
5 points

This homework invites you to acquaint yourself with another multiplicative function, and to see a little bit of the proof techniques in this area.

1. (a) Let $\omega(n)$ be the number of distinct prime divisors of $n > 1$, and let $\omega(1) = 0$. For example, $\omega(360) = 3$ since $360 = 2^3 \cdot 3^2 \cdot 5$. Show that $f(n) = 2^{\omega(n)}$ is a multiplicative function.

   (b) An integer is **square-free** if it is not divisible by the square of any prime. Show that $f(n)$ as defined above is the number of square-free positive divisors of n. For example, the positive divisors of 12 are $1, 2, 3, 4, 6, 12$, but only $1, 2, 3$ and 6 are square-free, so $f(12) = 4$.
   Hint: Use the fact that $f(n)$ is multiplicative to simplify the problem.