Math 255 - Spring 2022

Proofs, the gcd and the Euclidean algorithm

This contains proofs using concepts of the greatest common divisor and the Euclidean algorithm. This homework is worth 10 points.

1. (a) Let a and b be integers, not both of which are zero. Show that if $d = \gcd(a, b)$ and c is a common divisor of a and b, then $c|d$.

   (b) Use part (a) to show that if a and b are not both zero, then both definitions of greatest common divisor agree. (Recall that one definition says that the greatest common divisor of a and b is the common divisor d of a and b such that if c is also a common divisor of a and b, then $c \leq d$; the other definition says that the greatest common divisor of a and b is the common divisor $d$of a and b such that if c is also a common divisor of a and b, then c **divides** d.)

   (c) Prove that under the first definition of the greatest common divisor, $\gcd(0, 0)$ is not defined, and show that under the second definition of the greatest common divisor, $\gcd(0, 0) = 0$.

2. Prove that if c divides ab and $\gcd(a, c) = d$, then c divides db.

3. Prove that if $\gcd(a, b) = 1$ and $\gcd(a, c) = 1$, then $\gcd(a, bc) = 1$.