Math 255 - Spring 2022
Solving equations using primitive roots
10 points

This homework invites you to use primitive roots to solve congruences.

1. It is a fact that 3 is a primitive root modulo 17, and here is a table of discrete logs in base 3 modulo 17:

a	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$\log_3 a$	0	14	1	12	5	15	11	10	2	3	7	13	4	9	6	8

   Use this table to solve the following congruences:

   (a) $7^{5x} \equiv 3 \pmod{17}$

   (b) $5^{2x} \equiv 8^{3x} \pmod{17}$

2. (a) Find a primitive root r of 11.

   (b) For this primitive root r, compute $\log_r a$ for each $a \in (\mathbb{Z}/11\mathbb{Z})^\times$. Organize your answer in a nice table like I gave in problem 1.

   (c) Using your computations in part (b), solve the congruences

      i. $7x^3 \equiv 3 \pmod{11}$

      ii. $3x^4 \equiv 5 \pmod{11}$

      iii. $x^8 \equiv 10 \pmod{11}$