

Math 255 - Spring 2022  
Divisibility and gcd problems

This homework contains proofs using concepts of divisibility and the greatest common divisor. This homework is worth 15 points.

1. Use induction to show that

$$15 \mid (2^{4n} - 1) \quad \text{for } n \geq 1.$$

2. Prove that if  $a$  and  $b$  are both odd integers, then  $16 \mid (a^4 + b^4 - 2)$ .
3. Let  $a$  and  $b$  be integers, not both of which are zero. Prove that  $(a, b) = ((a, b), b)$ .
4. Let  $n$  and  $k$  be integers, not both of which are zero. Prove that  $(k, n + k) = 1$  if and only if  $(k, n) = 1$ .
5. Prove that if  $a \mid b$  and  $a > 0$  then  $(a, b) = a$ .